The Overall Balance Between Cooperation and Interference for a Class of Wireless Networks

Andrés Altieri, Leonardo Rey Vega, Pablo Piantanida and Cecilia G. Galarza

Abstract

This paper investigates the benefits of cooperation in large wireless networks with multiple sources and relays, where the nodes form an homogeneous Poisson point process. The source nodes may dispose of their nearest neighbor from the set of inactive nodes as their relay. Although cooperation can potentially lead to significant improvements on the asymptotic error probability of a communication pair, relaying causes additional interference in the network, increasing the average noise. We address the basic question: how should source nodes optimally balance cooperation vs. interference to guarantee reliability in all communication pairs. Based on the decode-and-forward (DF) scheme at the relays (which is near optimal when the relays are close to their corresponding sources), we derive closed-form approximations to the upper bounds on the error probability, averaging over all node positions. Surprisingly, in the small node-density regime, there is an almost binary behavior that dictates –depending on network parameters– the activation or not of all relay nodes.

Index Terms

Cooperative communication, interference, asymptotic error probability, outage probability, decode and forward, marked Poisson point processes.

This work was partially supported by DIGITEO-DIM No. 2010-33D (ACRON), the Peruilh grant of the UBA and project UBACYT 200200902003. The material in this paper was presented in part at the IEEE International Symposium on Information Theory, 2011 and submitted to the IEEE International Symposium on Information Theory, 2012.

L. Rey Vega and C. Galarza are with the Departments of Electronics (FIUBA) and CONICET, Buenos Aires, Argentina (e-mail: lrey@fi.uba.ar, cgalar@fi.uba.ar).

P. Piantanida is with the Department of Telecommunications, SUPELEC, 91192 Gif-sur-Yvette, France (e-mail: pablo.piantanida@supelec.fr).

I. Introduction

In order to cope with increasing traffic demands future wireless networks must employ strategies which provide an optimal use of resources such as bandwidth and power. Over the past decade there has been a great interest in cooperative networks [1] where relay nodes can be exploited as a means to increase throughput and reliability. Although the capacity of the single-relay channel [2] remains unsolved and its optimal coding scheme unknown, significant progress has been made in quantifying the performance gain obtained through cooperation. The information-theoretic research in this topic was mainly focused on simple networks with few nodes or fixed topologies where perfect channel state information (CSI) is available to all terminal nodes. Finding explicit capacity regions of large networks may be –if feasible– very hard. To tackle this limitation, spatial models employing tools from stochastic geometry and graphs provide a comprehensive framework for the analysis of large wireless networks with little interference management [3], [4]. In this setup, the interference between users is treated as noise [5] whose statistical properties depend on the particular spatial distribution of the nodes and fading realizations of the wireless paths.

In this paper we study upper bounds –in terms of the average *outage probability* (OP)– on the asymptotic error probability of large-scale decentralized wireless networks in which the sources are aided by nearby relays. More precisely, we consider a network formed by two types of clusters: source-relay-destination nodes, which use a cooperative transmission scheme and clusters with source-destination pairs which employ simple direct transmission (DT). We will concentrate in the special class of networks where the relays (when available) are in the proximity of their corresponding sources. The rationale behind this assumption comes from the fact that when a source attempts to transmit, the nodes in its vicinity are the ones which, with a high probability, become aware of the attempts of the source and can provide help. It is known that when a relay is located in the vicinity of the source, the *decode-and-forward* (DF) transmission scheme [1], [2] is capacity achieving. For this reason we will assume that the cooperative transmission scheme employed by a source-relay-destination triplet will be DF working in a full-duplex fashion. The network will be modeled as an independently marked *homogeneous Poisson point process* (HPPP), limited by the signal-to-interference ratio (SIR), where signal attenuation occurs both through path loss and slow fading. An outage event is

declared whenever the distribution of nodes and/or fading causes the target rate to be higher than the achievable rate. Hence the probability of these events (OP) becomes an upper bound on the asymptotic error probability of every pair of communicating nodes, which is the true performance metric of interest.

The central motivation behind the analysis of the asymptotic error probability of large wireless networks in the presence of relay nodes is to provide some understanding on the limitations and benefits of cooperation in such decentralized scenarios. In fact, the advantage of cooperation for an individual source-destination link was widely studied in the past years, addressing both theoretical and practical issues [1], [6], [7]. Although it is clear that relays can significantly improve the rate and reliability of a single source-destination pair, in large wireless networks where several source-relay-destination clusters are present the activation of relay nodes may drastically increase the overall interference. This occurs because as more relays are activated, other destinations and their corresponding relays will observe an increase in their interference levels. Thus, we see that although at a local level cooperation is beneficial, at a global scale it could be harmful. A deeper understanding of this balance between cooperation and interference generation is an important topic to be studied, because it could shed light on the usefulness of cooperation in interference-limited wireless networks of the type considered in this paper where the relays are in the proximity of their sources.

Nevertheless a complete unified answer to this delicate balance is out of the scope of this work and is certainly very difficult to obtain, mainly because it depends heavily on the way in which the sources select the relays, which could be influenced by local inhomogeneities in the network, local network knowledge available at nodes, etc. In this paper, we aim to answer this question for the simple network model introduced in Section II with the relays positioned near their corresponding sources. It is assumed that CSI is not available at the transmitting nodes which is often the case in decentralized wireless networks without feedback. Each candidate relay node decides whether to be active or not in a random manner. All the nodes make their decisions independently of each other, and of all network parameters. That is, the decision is simply made through a *Bernoulli* experiment with success probability given by p_r.

Regarding the spatial positions of the relay nodes, two different scenarios are investigated:

- The relays are located at a fixed relative distance with respect to their sources,
- Every source has its *nearest neighbor* (NN) (among some set of inactive nodes) associated

as a potential relay, which is obviously a reasonable assumption when using the DF scheme. The first case, although only of theoretical interest, is considered basically for mathematical tractability and because it gives some useful insights. In the second case the relays are randomly located, which means that in all clusters they are at different positions and hence averaging over all spatial positions of relays is needed to derive the OP. If a source is allowed to use its associated relay, cooperation will take place based on the DF scheme while if the relay is not activated, the source will use direct transmission (DT) to communicate with its intended destination. In this way the transmission scheme is a mixed one. To be fair, it should be mentioned here that there exist more sophisticated ways to decide whenever or not a relay node must be activated, e.g., by taking into account all effects mentioned above. The probabilistic model for activating relays that we assume is on one hand the simpler in mathematical terms to deal with, but on the other it still captures the balance between cooperation and interference generation via the parameter p_r. For instance, with $0 < p_r < 1$ each source-destination pair in the network is then able to resign some of it local performance (choosing not to use a relay and using DT) for the greater good (introducing less interference into the network). In this way, by studying the effect of p_r in the asymptotic error probability of a typical cluster, we will draw some conclusions about this important problem.

A. Related Work

Over the past years, performance gains of cooperative communications in relay networks were widely studied from an information-theoretic perspective. Since the seminal work of Cover-El Gamal [2], several contributions have been published on the subject. More recently, the emphasis was put in studying the performance of wireless relay channels where outage performance and ergodic rates of fading channels with Gaussian noise have been derived (see [1], [6]–[8] and the references therein). Among these valuable studies, the only impairments to the communication were due to additive Gaussian noise and fading, and very little attention was paid to the effect of the interference generated (or suffered) by the users. However, interference is probably the major impairment in wireless networks, specially in networks with little control and high mobility.

The study of the capacity of general wireless networks taking into account the interference generated by the different users was pioneered by the seminal work of Gupta-Kumar [9], where the concept of transport capacity and fundamental scaling laws on the network throughput were

obtained considering only point-to-point coding. In [10] multiuser achievability regions were obtained and it was shown that for some special wireless networks significantly better scaling laws on the network throughput, with respect to the case in [9], are possible. Further progress was done in [11] where new scaling laws are derived using coherent multistage relaying with interference subtraction and in [12], where extensions to fading channels were obtained.

In all the above works, the achievability of the scaling laws derived is obtained for extremely regular networks which is seldom the case in practice. In that sense, stochastic geometry and point processes [13], [14] are not only elegant mathematical frameworks but also seem to be useful tools to deal with more realistic network models, where the spatial position of nodes and the effect of interference can be incorporated in a probabilistic manner [3]. Although several types of point processes can be used to model different kind of networks, it is the HPPP which has received more attention. Although other types of point processes could provide more realistic models [15], the extended use of HPPP comes from the possibility to obtain simple closed form results in several cases of interest. The quantity called transmission capacity (TC) was introduced in [16] in order to include outage probability constraints in the scaling behavior. Several results have been obtained, through the use of the TC, for several practical situations, as multiple input-multiple output capable users in wireless networks [17], decentralized power control [18], etc. (for a review of several other important results we refer the reader to see [4] and references therein).

B. Main Contributions

The main contributions are twofold. First, tight approximations for the OP, which lend themselves to analysis, are derived. Emphasis is put on the small node-density regime (SNDR), where the density of active transmitters is low and which is a natural network operating point. Secondly, we determine the relay activation probability which provides the optimal balance between the reliability gain obtained from cooperation and the overall interference introduced in the network. From a different perspective, this addresses also the question whether or not to cooperate. The central conclusion we draw here is that, under quite general assumptions and for a large range of network parameters, the optimal activation probability p_r is either 0 or 1. This implies that for all clusters in the network it is best to fully cooperate or not at all. In other words, there is no overall gain if some clusters are permitted to cooperate and others not.

The paper is organized as follows. In section II a general and a mathematical description of the network model are presented. We also discuss the DF scheme and its achievable rate in the assumed network model. In Section III we present results corresponding to the OP performance when only one cluster has a relay, and the other users in the network use only DT. Although several results exists for the outage probability performance in fading relay channels under Gaussian noise [6], [7], there are no results, at least to our knowledge, where the communication impairments come also from a network of interferers. It will be shown that even in this simpler case the expression of the OP cannot be obtained in closed form and some upper bounds, valid when the relay is close to the source or when the outage probability is sufficiently small, are provided. In Section IV the more important case where there may be a relay available for each user in the network is treated. Several interesting situations are treated: collocated relays, fixed but not collocated relays, and random relays distributed with the NN distribution. Network parameter regions where the optimal p_r shows a binary behavior are characterized. Moreover, the network parameter regions where $p_r = 1$ is optimal are identified. Numerical simulations are also presented in order to validate the derived analytical results. In section V a short discussion on the possible use of local CSI at the relays for more elaborated activation schemes is presented. Section VI provides some concluding remarks. Finally, several longer mathematical proofs are grouped together by section and deferred to the appendices.

Notation

We will denote $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{R}^2$, the real numbers, complex numbers and the real plane respectively. The canonical euclidean norm will be denoted as $\|\cdot\|$. With $(\cdot)^*$ we denote complex conjugation and with $\Re(\cdot)$ the real part of complex number. $\mathbb{P}_X\left\{\cdot\right\}$ and $\mathbb{E}_X\left[\cdot\right]$ denote probability and expectation operators with respect to the distribution of the random variable X. In a similar way $\mathbb{P}_{X|Y}\left\{\cdot|Y\right\}$ and $\mathbb{E}_{X|Y}\left[\cdot|Y\right]$ denote probability and expectation operators with respect to the distribution of the random variable X conditioned to random variable Y. With $\mathcal{A}^c$ we denote the complement of the set $\mathcal{A}$. We shall also use the big O notation: $f(x) = O\left(g(x)\right)$ as $x \to x_0$ if there exists $M > 0$ and such that $|f(x)| \leq M|g(x)|$ is some neighborhood of x_0.

II. General considerations and network model

Consider a spatial network model in $\mathbb{R}^2$ in which source nodes generate messages and attempt to transmit them to intended destinations, either through a direct link, in which case the destinations only receive symbols from their source, or using others nodes that act as relays. Every relay aids a single source node and is assumed to be located in its vicinity, acting as a secondary full-duplex transmitter sharing the same time slots and frequency band. This setup allows the nodes to be grouped into *clusters* formed by a source-destination pair or by a source-relay-destination triplet, if the source has an associated relay, as shown in Fig. 1.

The network is constructed in a two-stage fashion starting from the a set of nodes Φ. In the first stage, the source nodes are selected from the set Φ and the rest is left without a specific role. At the end of this stage, the original set of nodes Φ is divided into two disjoint sets, namely, the set of sources Φ_s and the set of inactive nodes Φ_{in} which do not have a role yet. We will assume that the set of all nodes Φ forms a homogeneous Poisson point process (HPPP) of intensity λ [14], and that the role in the first stage is assigned through independent marking. This means that at the end of the first stage, the set of nodes Φ can be thought as the superposition of two independent HPPPs, namely (Φ_s, Φ_{in}), with intensities $(\lambda_s, \lambda_{in})$, such that $\lambda = \lambda_s + \lambda_{in}$. It should be clear that this can also be thought as that the medium access control (MAC) protocol working in the modeled network is a slotted ALOHA [3], such that at each slot every node decides whether to transmit or not. It is assumed that the intended destinations for the sources nodes are not part of the process Φ.

At the second stage every source node will be associated or not to an active relay node in its vicinity. Hence the problems to be dealt with are how such decision should be made and how a relay should be chosen among a set of suitable candidates. It is clear that an adequate decision rule for relay activation should take into account local criteria, to satisfy the clusters needs, and global ones, to achieve certain goals such as network throughput. As a matter of fact, if all relay nodes had full CSI of all (including destination) nodes involved in the network, they could construct a "global" consensus to evaluate (depending on fading, node proximities, etc.) the best way to associate relays to source-destination pairs and at the same time to avoid causing too much interference to each other. Nevertheless, this does not apply to our setting since we shall assume only little –local– information is available at relays, no information about the

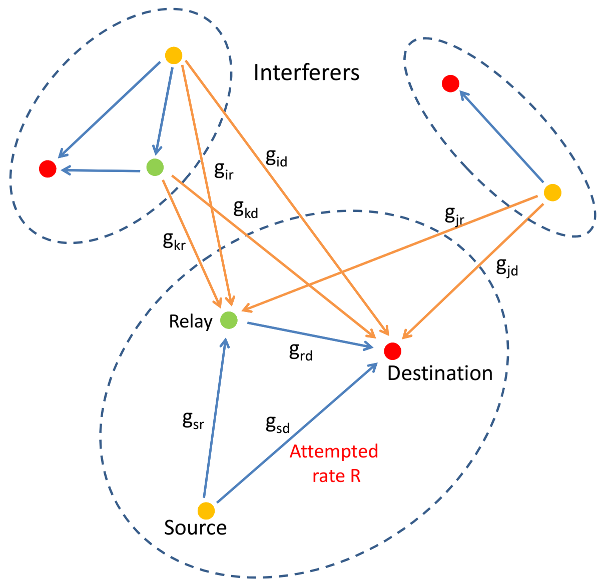


Fig. 1. General additive relay channel. g_{sr}, g_{rd}, and g_{sd} are the channel gains between source and relay, relay and destination and source and destination, respectively. The interferers i, j and k and their corresponding channel gains toward the relay and the destination are also shown.

destination nodes and no CSI is available at the source nodes. Although these assumptions may appear to be restrictive, they are quite realistic in decentralized wireless networks. For instance, we assume that the relays solely decide their activation randomly and with the same probability and independently of all network parameters[1]. From a mathematical standpoint, this means that each node of the set of sources Φ_s is assigned with a *Bernoulli* mark of success probability p_r, indicating whether the relay associated to a source is activated or not[2]. It will be assumed that the corresponding sources remain oblivious of the relay decisions. Regarding the relay selection, we shall consider two situations:

- For a source node at position x, the relay is positioned at a fixed relative position from the source denoted by $r = x + k$, where k is a fixed two-dimensional vector.
- The relay is chosen by the source from the set of unassigned nodes Φ_{in}.

The first case is considered for mathematical tractability and because some useful insights about the general problem can be obtained. With respect to the second case, it is reasonable to assume that relays will choose their sources using the scarce local knowledge of the network

[1]In section V we briefly consider the case in which the relays use local CSI in order to decide their activation.

[2]Note that this model can also handle the possibility that a source does not have a potential relay available.

they possess. The relay assignment is a very complex issue, since on one hand a one to one correspondence between the sources and their relays has to be established, and on the other hand this correspondence must be established so that the pairs are appropriately located for effective cooperation. Since the focus will be on networks where the relays are in a vicinity of their sources, we propose that each source will attempt to associate its nearest neighbor (NN) from the set Φ_{in}. This assignment rule does not avoid the possibility of two sources being associated to the same node as a relay since the same unassigned node could be the closest neighbor of one or more sources. Nevertheless, assuming a reasonably small node-density functioning regime the density of active sources (λ_s) will be much smaller than that of the potential relays (λ_{in}) and the probability of conflicts will be small.

With these simplifications, the existence and location of all the relays become independent for each source and we can simply include the position of the NN of each source as an independent mark distributed with the NN probability distribution. In this way, we do not have to deal with conflicts and a possible dependence between the relay assignments in different source nodes, simplifying the model considerably.

As a final remark, it is important to note that in each cluster we have a mixed scenario, i.e. with probability p_r a cooperative scheme is used while DT will be implemented with probability $(1 - p_r)$. Notice that the relay activation probability p_r becomes the crucial parameter which dictates the balance between cooperation and the aggregate interference in the network. As p_r increases a greater proportion of clusters enjoy the benefit of cooperation, supporting a greater rate or having a better quality of service. But at the same time more interference is introduced in the network. Therefore, the main question would be: which is the optimal activation probability at a global scale? From a practical point of view, this question is relevant because its answer would increase our understanding of the limits of cooperation in interference-limited networks. A priori, the answer is not obvious and in the rest of this paper we will try to give some insights about it, at least for our simplified model.

A. Mathematical Description

Based on our simplifying hypothesis described above, we now provide a more rigorous description of the network model. The network is modeled as an independently marked HPPP:

$$\tilde{\Phi}_s = \{(x_i, \varepsilon_{x_i}, k_i, h_{x_i r}, h_{x_i d}, h_{k_i r}, h_{k_i d})\}\,. \tag{1}$$

satisfying the following requirements:

- The positions of the sources constitute the HPPP $\Phi_s = \{x_i\}$ of intensity λ_s.
- The *Bernoulli* random variable (RV) ε_{x_i} indicates that source x_i uses a relay, according to an event with probability p_r.
- The relay associated with the source at x_i is located at $x_i + k_i$, where k_i is a degenerate RV or with the NN from Φ_{in} distribution. All source-destination distances are D.
- All nodes transmit with unit power while the power received at y by a transmitter at x is $|h_{xy}|^2 l_{xy}$ where $l_{xy} = \|x - y\|^{-\alpha}$ ($\alpha > 2$) is the usual path loss function and $|h_{xy}|^2$ is the power gain of Rayleigh fading[3] with unit mean. This is equivalent to saying that h_{xy} are complex, circular [19], zero-mean Gaussian RVs.
- An additional source with the same marks as the others, independent of the point process $\tilde{\Phi}_s$ and with its destination at $d = [D, 0]$, is added at the origin. The position of the relay for this source node will be r (distributed as a degenerate RV or distributed with the NN distribution). The coefficients $|h_{sr}|^2$, $|h_{rd}|^2$ and $|h_{sd}|^2$ model the source-relay, relay-destination, and source-destination fading coefficients of this cluster, respectively. This cluster is independent of the point process $\tilde{\Phi}_s$. Slyvniak's Theorem [13], [14] guarantees that the study of this cluster's behavior will be representative of the behavior of any other similar cluster in the network and hence it can be considered as a "typical cluster".
- $h_{x_i r}$, $h_{k_i r}$ and $h_{x_i d}$, $h_{k_i d}$ model the fading gains between each source and its relay and the relay of the source at the origin, and between each source and its relay and the destination of the source at the origin, respectively. Thus the gains shown in Fig. 1 correspond to $|g_{sr}|^2 = |h_{sr}|^2 l_{sr}$, $|g_{sd}|^2 = |h_{sd}|^2 l_{sd}$ and $|g_{rd}|^2 = |h_{rd}|^2 l_{rd}$.

It will be assumed that the HPPP $\tilde{\Phi}_s$ remains fixed during the transmission time.

Remark 2.1: The path loss l_{xy} function is a very accurate model of wireless power propagation when the distance between the points x and y is sufficiently large (far-field approximation). However, it is not a good model when x is close to y because of the existence of a singularity which makes it unbounded. A more suitable choice would be [20]:

$$l_{xy} \equiv \frac{1}{1 + \|x - y\|^{\alpha}}, \tag{2}$$

[3]In this paper we focus on Rayleigh fading but, the results can be extended to the case of Nakagami fading [17].

which avoids the singularity and has the desired behavior when $\|x-y\|$ is large. In despite of this, the above function would increase the analytical complexity with small benefits when the density of active transmitters λ_s is small [4], [21] (typical operation regime).

We shall assume that at each position y in the space a background complex, circular and zero-mean Gaussian noise Z_y is present. This noise has variance $1/\mathrm{SNR}$ and it is statistically uncorrelated in time. There is no need in specifying any spatial correlation on such background noise. Then, by conditioning on the marked HPPP, the signals received at the relay and destination –associated with the source node at the origin– can be written as:

$$Y_r = \frac{h_{sr}X_s}{\|r\|^{\frac{\alpha}{2}}} + \underbrace{\sum_{i:x_i\in\Phi_s}\left(\frac{h_{x_ir}X_{x_i}}{\|x_i-r\|^{\frac{\alpha}{2}}} + \varepsilon_{x_i}\frac{h_{k_ir}X_{k_i}}{\|x_i+k_i-r\|^{\frac{\alpha}{2}}}\right)}_{\tilde{Z}_r} + Z_r, \tag{3}$$

$$Y_d = \frac{h_{sr}X_s}{\|r\|^{\frac{\alpha}{2}}} + \frac{h_{rd}X_r}{\|r-d\|^{\frac{\alpha}{2}}} + \underbrace{\sum_{i:x_i\in\Phi_s}\left(\frac{h_{x_id}X_{x_i}}{\|x_i-d\|^{\frac{\alpha}{2}}} + \varepsilon_{x_i}\frac{h_{k_id}X_{k_i}}{\|x_i+k_i-d\|^{\frac{\alpha}{2}}}\right)}_{\tilde{Z}_d} + Z_d, \tag{4}$$

where, for shortness, we have dropped the dependence of the signals on the messages to be transmitted and the discrete time indices for the block codewords. We have denoted with (X_s, X_r) the complex, circular and zero-mean Gaussian signals at the source and relay (if any is associated) for the cluster at the origin, and (X_{x_i}, X_{k_i}) the corresponding signals for the other clusters in the network. Source and relay nodes in each cluster are assumed to not perform any joint decoding of their own messages with the messages of other clusters in the network. Therefore, they will simple take those interference signals as pure noise. Therefore, the relay and the destination associated with the source at the origin see aggregate interferences $\tilde{Z}_r$ and $\tilde{Z}_d$, respectively, yielding the following lemma.

Lemma 2.1: Let $\alpha > 2$, then for almost all realizations of the point process $\tilde{\Phi}_s$, the aggregate interferences $\tilde{Z}_r$ and $\tilde{Z}_d$ are zero-mean complex circular Gaussian variables whose conditional variances are respectively given by

$$I_r = \sum_{i:x_i\in\Phi_s}\left[\frac{|h_{x_ir}|^2}{\|x_i-r\|^{\alpha}} + \varepsilon_{x_i}\left(\frac{|h_{k_ir}|^2}{\|x_i+k_i-r\|^{\alpha}} + \frac{2\Re\left\{h_{x_ir}h^*_{k_ir}\rho\right\}}{\|x_i-r\|^{\frac{\alpha}{2}}\|x_i+k_i-r\|^{\frac{\alpha}{2}}}\right)\right], \tag{5}$$

$$I_d = \sum_{i:x_i\in\Phi_s}\left[\frac{|h_{x_id}|^2}{\|x_i-d\|^{\alpha}} + \varepsilon_{x_i}\left(\frac{|h_{k_id}|^2}{\|x_i+k_i-d\|^{\alpha}} + \frac{2\Re\left\{h_{x_id}h^*_{k_id}\rho\right\}}{\|x_i-d\|^{\frac{\alpha}{2}}\|x_i+k_i-d\|^{\frac{\alpha}{2}}}\right)\right]. \tag{6}$$

Proof: The proof follows from the fact that when $\alpha > 2$, I_d and I_r are finite for almost every realization of $\tilde{\Phi}_s$. This can be shown using the Laplace functional of $\tilde{\Phi}_s$ [13], [22], and the following functions

$$f(x, \varepsilon, k, h_1, h_2, h_3, h_4) = \frac{|h_1|^2}{\|x-r\|^\alpha} + \varepsilon \left(\frac{|h_3|^2}{\|x+k-r\|^\alpha} + \frac{2\Re\{h_1 h_3^* \rho\}}{\|x-r\|^{\frac{\alpha}{2}}\|x+k-r\|^{\frac{\alpha}{2}}} \right) \quad (7)$$

and $g(x, \varepsilon, k, h_1, h_2, h_3, h_4)$ defined in an similar form. As the signals (X_{x_i}, X_{k_i}) and (X_{x_j}, X_{k_j}) are Gaussian and independent when $i \neq j$, from equation (13) below it can be shown that the partial sums (through a proper enumeration of the points of the particular realization of $\tilde{\Phi}_s$) in $\tilde{Z}_r$ and $\tilde{Z}_d$ are Gaussian with variances given by the corresponding partial sums in I_d and I_r. Thanks to the finiteness of I_d and I_r, this family of Gaussian distributions have the *tightness* property (Theorem 25.10 in [23]) which allow us to obtain the desired result. ■

B. Problem Statement and Bounds on the Asymptotic Error Probability

Our goal will be to study the asymptotic error probability performance of cooperative communication schemes, which could make use of the presence of potential relay nodes. In order to do that let us now define the notion of code which is the same for any cluster in the network, similarly to that of point-to-point codes in [5]:

Definition 2.1 (single-relay code): A single-relay code-$\mathcal{C}_n(n, R, M_n)$ for a set $\{1, \dots, M_n\}$ of uniformly distributed messages W consists of a set of randomly and independently generated complex Gaussian codewords X_s^n, each according to n i.i.d. draws of a Gaussian RV of unit variance, a decoder mapping $\hat{W} : \mathbb{C}^n \longmapsto \{1, \dots, M_n\} \cup \{\mathcal{E}\}$, and a sequence of relay mappings $f_t : \mathbb{C}^{t-1} \longmapsto \mathbb{C}$ constrained to produce i.i.d. complex Gaussian RVs of unit variance, for $t = \{1, \dots, n\}$. The smallest asymptotic average (over all random parameters) probability of error of, for example, the source-destination pair at the origin and which will be representative of any source-destination pair in the network, is given by

$$\bar{P}_e(R) \equiv \inf_{\mathcal{C}_n} \left\{ \limsup_{n\to\infty} \mathbb{P}_\Theta^{(n)}(W \neq \hat{W}|\mathcal{C}_n) \,\Big|\, \liminf_{n\to\infty} \frac{1}{n}\log M_n \geq R \right\},$$

where Θ condense all the randomness in the model:

$$\Theta = \left\{ \tilde{\Phi}_s, h_{sr}, h_{rd}, h_{sd}, r, \varepsilon_0 \right\}. \quad (8)$$

Notice that the source is unaware of the instantaneous interference, path loss attenuation and fading coefficients involved, which implies that the error probability cannot be made arbitrary small with the code-length.

The asymptotic error probability respect to a source node located at the origin can be upper bounded using any code $\mathcal{C}_n$ as follows:

$$\bar{P}_e(R) \leq \inf_{\mathcal{O}(R)\in\sigma(\Theta)} \left[\mathbb{P}_\Theta\left\{\mathcal{O}(R)\right\} + \limsup_{n\to\infty} \mathbb{P}_\Theta^{(n)}\left\{W \neq \hat{W}|\mathcal{C}_n, \mathcal{O}^c(R)\right\}\right] \tag{9}$$

where $\sigma(\Theta)$ is the σ-algebra generated by Θ and $\mathcal{O}(R)$ denotes any outage event. By choosing adequately the outage event $\mathcal{O}(R)$ for a given code $\mathcal{C}_n$, it turns out that the second term on the right-hand side of (9) can be made arbitrary small. That is, for any $\epsilon > 0$:

$$\limsup_{n\to\infty} \mathbb{P}_\Theta^{(n)}\left\{W \neq \hat{W}|\mathcal{C}_n, \mathcal{O}^c(R)\right\} \leq \epsilon. \tag{10}$$

In this way, fixed a rate R, the asymptotic error probability $\bar{P}_e(R)$ is dominated by the outage probability (OP) $\mathbb{P}_\Theta\left\{\mathcal{O}(R)\right\}$ of the corresponding achievable rate. The OP is a useful performance metric which was extensively employed to characterize performance in a Poisson field of interferers, jointly with the associated metric of *transmission capacity* (see [4], [16], [22] and the references therein).

The average error probability of any single-relay code-$\mathcal{C}_n(n, R, M_n)$ defined as above can be lower bounded as follows [24]:

$$\begin{aligned}\bar{P}_e(R) &\geq \inf_{P_{X_r X_s}} \mathbb{P}_\Theta\{R > \min\left\{I(X_s;Y_rY_d|X_r), I(X_sX_r;Y_d)\right\}\} \\ &\geq \inf_{P_{X_r X_s}} \mathbb{P}_\Theta\{R > I(X_sX_r;Y_d)\}\,. \end{aligned} \tag{11}$$

The rationale for the second inequality comes from the fact that for the class of networks considered here, where the relays are in a vicinity of the sources, the limiting term in the cut-set bound is $I(X_sX_r;Y_d)$.

In our context of Poisson networks we shall also consider the scaling behavior of the error probability with the density of interferers. We have the following definition:

Definition 2.2 (small node-density regime): We define the small node-density regime (SND) as the situation in which $\lambda_s\pi D^2 \to 0$ characterized by the following metric:

$$\kappa(R) \equiv \lim_{\lambda_s\pi D^2\to 0} \frac{\bar{P}_e(R)}{\lambda_s\pi D^2}. \tag{12}$$

As $\lambda_s \pi D^2$ is the average number of active nodes in circle of radius D, $\kappa(R)$ gives the asymptotic behavior of the smallest probability of error for a code with rate R, when the number of active users in disk of radius D centered at the destination goes to zero. In other words, $\kappa(R)$ measures the reliability of the communication when the closest point to the destination is the source that intends to transmit to it, and the destination does not see the interference from other nearer nodes which would have stronger channels because of their reduced path loss. In this way $\kappa(R)$ is a metric, similar to the standard notion of diversity gain [25] used for fading Gaussian channels, that measure the robustness against the interference generated by the network.

C. Achievable Bounds on the Asymptotic Error Probability

The main coding strategies for relay networks were introduced in the seminal work by Cover-El Gamal [2]. There have been three dominant relaying paradigms: decode-and-forward (DF), compress-and-forward (CF), and amplify-and-forward (AF). Decode-and-Forward (DF) scheme allows the relay to decode the messages sent by the source, re-encode them, and forward them to the destination. In the special case where the memoryless relay channel is *physically degraded* the achievable rate using DF is in fact the capacity. However, in the general case, it turns out that, depending on the channel parameters, there is no a unique scheme maximizing the rate. In general, roughly speaking, if the source-relay channel has less noise than the source-destination one, the relay can perfectly decode the messages intended to the destination without introducing any bottleneck in the information flow. This does not hold true for all cases since the imposition of full decoding at the relay could be a strong one. Although some variants as partial-decode-and-forward (P-DF) [2] may ameliorate this inconvenience, we will focus on the DF scheme. The potential improvement of P-DF essentially relies on a careful optimization of the code at the encoder, which cannot be correctly done in our setting due to the lack of CSI at the source. Moreover, our interest will be in networks where the relays are located in some vicinity of their corresponding sources, generally closer to them than to the associated destinations. In that situation, as path loss has a major role in channel quality, assuming that the distance source-to-relay is smaller in average than that of source-to-destination, the DF scheme should not introduce significant penalties with respect to other schemes.

There exist several coding and decoding strategies based on block-Markov superposition

encoding that achieve the same DF rate. In [2], cooperative strategies use *irregular encoding*, *random binning* and *successive decoding* at the destination. However, this method is not well-suitable for the present scenario. Helpfully, the same rate can be derived with *regular encoding* and *sliding-window decoding* [26] at the destination. Another cooperative scheme that permits to achieve same rate is *regular encoding* and *backward decoding* [27], [28]. We assume that transmissions are done based on regular encoding and sliding-window decoding [1] with Gaussian signaling. The n-length random codewords at each source and its associated relay is written as

$$X_s^n(w_{i-1}, w_i) = \sqrt{(1-|\rho|^2)}\tilde{X}_1^n(w_i) + \rho\tilde{X}_2^n(w_{i-1}),\ X_r^n(w_{i-1}) = \tilde{X}_2^n(w_{i-1}), \tag{13}$$

for messages $w_i \in \{1, \dots, 2^{nR_{DF}}\}$ with $w_0 = w_{B+1} = 1$ and each block $i = \{1, \dots, B\}$, where $\tilde{X}_1$ and $\tilde{X}_2$ are independent complex, circular Gaussian RVs with unit variance and ρ is the correlation coefficient between source and relay signals X_s and X_r. Then, conditioned on a particular realization of $\tilde{\Phi}_s$, we can guarantee that for the relay channel associated with the source located at the origin the following rate is achievable using the DF scheme [2]:

$$R_{DF} = \max_{\rho\in\mathbb{C},|\rho|\leq 1} \min \left\{ \mathcal{C}\left(\frac{|h_{sr}|^2 l_{sr}(1-|\rho|^2)}{I_r + 1/\text{SNR}} \right), \mathcal{C}\left(\frac{|h_{sd}|^2 l_{sd} + |h_{rd}|^2 l_{rd} + 2\sqrt{l_{sd} l_{rd}}\Re\left(\rho h_{sd} h_{rd}^*\right)}{I_d + 1/\text{SNR}} \right) \right\}, \tag{14}$$

where $\mathcal{C}(u) = \log_2(1+u)$.

Define the outage event when the relay is present as $\mathcal{O}_1(R) = \{\mathcal{A}(R,\rho) \cup \mathcal{B}(R,\rho)\}$ with

$$\mathcal{A}(R,\rho) = \left\{ |h_{sr}|^2 l_{sr}\left(1-|\rho|^2\right) < T(I_r + 1/\text{SNR}) \right\},$$
$$\mathcal{B}(R,\rho) = \left\{ |h_{sd}|^2 l_{sd} + |h_{rd}|^2 l_{rd} + 2\sqrt{l_{sd} l_{rd}}\Re(\rho h_{sd} h_{rd}^*) < T(I_d + 1/\text{SNR}) \right\},$$

for which condition (10) holds true. The event $\mathcal{A}(R,\rho)$ means that the relay is in outage while $\mathcal{B}(R,\rho)$ means that the destination is in outage while source and relay cooperate. $T = 2^R - 1$ and ρ is the complex correlation coefficient between the transmitted source and relay symbols. The error probability when the source at the origin has an associated relay is bounded by:

$$\begin{aligned} \bar{P}_e(R|\varepsilon_0 = 1) \leq \mathbb{P}_{\text{out,DF}}(R) &= \inf_{\rho\in\mathbb{C},|\rho|\leq 1} \mathbb{P}_\Theta\left\{\mathcal{O}_1(R)|\varepsilon_0 = 1\right\} \\ &= \inf_{\rho\in\mathbb{C},|\rho|\leq 1} \mathbb{P}_\Theta\left\{\mathcal{A}(R,\rho) \cup \mathcal{B}(R,\rho)|\varepsilon_0 = 1\right\}. \end{aligned} \tag{15}$$

We also define the outage event $\mathcal{O}_0(R) = \mathcal{A}_{DT}(R)$ for the case in which there is no relay and thus the source simply uses DT with

$$\mathcal{A}_{DT}(R) = \left\{ \frac{|h_{sd}|^2 l_{sd}}{I_d + 1/\text{SNR}} < T \right\}, \tag{16}$$

for which again condition (10) holds true. For the source at the origin and without relay, the error probability is upper bounded by

$$\bar{P}_e(R|\varepsilon_0 = 0) \leq \mathbb{P}_{\text{out,DT}}(R) = \mathbb{P}_\Theta\left\{\mathcal{O}_0(R)|\varepsilon_0 = 0\right\} = \mathbb{P}_\Theta\left\{\mathcal{A}_{DT}(R)|\varepsilon_0 = 0\right\}, \tag{17}$$

and for the general case with mixed situations, the upper bound writes as

$$\bar{P}_e(R) \leq \mathbb{P}_{\text{out,mix}}(R) = \inf_{0\leq p_r\leq 1} \inf_{\rho\in\mathbb{C},|\rho|\leq 1} p_r\mathbb{P}_\Theta\left\{\mathcal{O}_1(R)|\varepsilon_0 = 1\right\} + (1-p_r)\mathbb{P}_\Theta\left\{\mathcal{O}_0(R)|\varepsilon_0 = 0\right\}. \tag{18}$$

Remark 2.2: It is important to mention that the events $\mathcal{A}(R,\rho)$ and $\mathcal{B}(R,\rho)$ are the outage events only if regular encoding with sliding-window decoding or regular encoding with backward decoding are used. If irregular encoding with successive decoding is used we should consider another event regarding the estimation by the destination of the binning index sent by the relay, taking into account the rate of that transmission which is different from R.

Remark 2.3: The DF scheme with backward or sliding-window decoding at the destination are *oblivious* [8] to the potential absence of the relay. That is, in the absence of the relay (and without the source knowledge of that event), the DF scheme does not degrade the performance with respect to DT. In this way, the coding scheme employed by the source can always be the same (equation (13)) independently of the activation or not of the associated relay.

Remark 2.4: The rate R_{DF} does not depend on the correlation between the noises or interferences at relay and destination. This is true because the correlation between received signals at the relay and the destination becomes irrelevant when full decoding at the relay is imposed. As a matter of fact, this is not the case for the CF and AF schemes where the correlation between the noises can increase or decrease the corresponding achievable rate [29].

III. A SINGLE RELAY CHANNEL AND MULTIPLE SOURCES

In this section we will consider the case when only the source at the origin has a relay associated with probability one and the other sources in the network transmit without the help of a relay. The goal of this section is to provide a quantitative analysis of how cooperation improves performance in a single cluster and how interference reduces its benefits. In the case in which only Gaussian background noise and Rayleigh fading is considered (without interference) very interesting gains have been observed in terms of OP [6] [7]. The setup with interference in which only one source has a relay can be considered as a best case scenario from the point

of view of this source, and it will give insight to the general problem of DF performance in interference-limited networks. Although this case may seem like a direct extrapolation of the case with background noise, in this scenario we must average over all possible configurations of interfering nodes, considering numerous situations in which communications are severely impaired due to the presence of heavy interference. As we will see, this results in performance gains which are not as large as in the case in which only fading and noise are considered.

In the following we shall consider that there is no background Gaussian noise (SNR $\to \infty$), and that the only impairment is the interference generated by the other nodes in the network. The motivation for this is twofold. On one hand, it permits us to focus on the more important effect of the aggregate interference, which could be several times greater than the background noise [22]. On the other hand, not considering the background noise will lead to more compact expressions which are easier to analyze. Nevertheless, the results obtained could be modified without too much effort in order to include the effect of background Gaussian noise.

We start by deriving the OP in the case in which the relay of the cluster at the origin is at a fixed known location (Theorem (3.1)). Although a closed form expression can be derived in terms of the joint Laplace transforms of the interferences at the relay and at the destination, it involves certain integrals which cannot be evaluated in closed form [30]. Therefore we find upper bounds on the OP which are tight when the OP is small (Theorem 3.2). In addition, we study the optimal correlation coefficient ρ between the symbols transmitted by the source and the relay, and find that in the SND regime its value is zero. Afterwards we study the case in which this single relay is randomly distributed according to the nearest neighbor distribution of the source. Direct evaluation of the OP in this case can not be obtained through exact expressions, but starting from the bounds previously obtained for the fixed relay we develop upper bounds for the OP in this scenario, which are tight when the OP is small (Theorem 3.3). This allows for a characterization of the OP and a comparison to DT in a more realistic scenario.

A. *Relay at a Fixed Position*

The outage probability when the source at the origin is the only one having a fixed relay at a known position r can be written as[4]:

$$\mathbb{P}_{\text{out,DF}}(R) = \inf_{\rho\in\mathbb{C},|\rho|\leq 1} \left\{1 - \mathbb{P}_{\Theta}\left\{\mathcal{A}^c(R,\rho)\cap\mathcal{B}^c(R,\rho)\right\}\right\}. \tag{19}$$

In the following, in order to simplify the expressions, we will drop the inf operation and it will be assumed implicitly. From the definitions of $\mathcal{A}(R,\rho)$ and $\mathcal{B}(R,\rho)$ we can therefore write:

$$\mathbb{P}_{\text{out,DF}}(R)=1-\mathbb{P}_{\Theta}\left\{|h_{sr}|^2\geq\frac{TI_r}{l_{sr}\left(1-|\rho|^2\right)},\frac{|h_{sd}|^2 l_{sd}+|h_{rd}|^2 l_{rd}+2\sqrt{l_{sd}l_{rd}}\Re(\rho h_{sd}h_{rd}^*)}{I_d}\geq T\right\}. \tag{20}$$

It is interesting to mention that the OP has two different expressions according to the relay position r and the correlation coefficient ρ of the symbols transmitted by the source and the relay. The following Theorem, which deals with the expression of the OP in this setup, includes both expressions. However, as we shall see in what follows, working with only one of them is enough for characterizing the OP behavior.

Theorem 3.1 (OP for a single fixed relay): The outage probability for the DF protocol when only the cluster at the origin has a relay and $||r-d||\neq D$ or $\rho\neq 0$ is:

$$\mathbb{P}_{\text{out,DF}}(R) = 1-\frac{\mu_2}{\mu_2-\mu_1}\mathcal{L}_{I_d,I_r}(T/\mu_2,T/\mu_3)+\frac{\mu_1}{\mu_2-\mu_1}\mathcal{L}_{I_d,I_r}(T/\mu_1,T/\mu_3), \tag{21}$$

where:

$$\mu_1 = \frac{(l_{sd}+l_{rd})-\left((l_{sd}-l_{rd})^2+4l_{sd}l_{rd}|\rho|^2\right)^{1/2}}{2}, \tag{22}$$

$$\mu_2 = \frac{(l_{sd}+l_{rd})+\left((l_{sd}-l_{rd})^2+4l_{sd}l_{rd}|\rho|^2\right)^{1/2}}{2}, \tag{23}$$

$$\mu_3 = l_{sr}\left(1-|\rho|^2\right), \tag{24}$$

and:

$$\mathcal{L}_{I_d,I_r}\left(\omega_1,\omega_2\right) := \mathbb{E}_{\tilde{\Phi}_s}\left[e^{-(\omega_1 I_d+\omega_2 I_r)}\right],\ \omega_1,\omega_2\in\mathbb{C}, \tag{25}$$

[4]Since in this case only the source at the origin has a relay associated at position r with probability one, the randomness in the model is given by $\Theta=\left\{\tilde{\Phi}_s, h_{sr}, h_{rd}, h_{sd}\right\}$. Similarly, the marks in $\tilde{\Phi}_s$ associated with the relays are not considered, and the interference at the destination associated with the source at the origin can be written as:

$$I_d = \sum_{i:x_i\in\Phi_s}\frac{|h_{x_i d}|^2}{\|x_i-d\|^\alpha},$$

with a similar expression for the interference at the relay's position.

with $\Re\{\omega_1\}, \Re\{\omega_1\} > 0$ is the joint Laplace transform of the interference at the relay and at the destination. In addition, when $||r-d|| = D$ and $\rho = 0$, we have $\mu_1 = \mu_2$ and the outage probability is:

$$\mathbb{P}_{\text{out,DF}}(R) = 1 + \frac{T}{\mu_1} \frac{d\mathcal{L}_{I_d,I_r}(\omega_1, T/\mu_3)}{d\omega_1}\bigg|_{\omega_1=T/\mu_1} - \mathcal{L}_{I_d,I_r}(T/\mu_1, T/\mu_3). \tag{26}$$

Proof: Defining:

$$V := |h_{sd}|^2 l_{sd} + |h_{rd}|^2 l_{rd} + 2\sqrt{l_{sd} l_{rd}} \Re(\rho h_{sd} h_{rd}^*), \tag{27}$$

and considering that h_{sr} and V are independent of each other and of Φ_s we have that:

$$\mathbb{P}_{\text{out,DF}}(R) = 1 - \mathbb{E}_{\tilde{\Phi}_s}\left[\mathbb{P}_{h_{sr},V|\tilde{\Phi}_s}\left\{|h_{sr}|^2 \geq \frac{TI_r}{\mu_3}, \frac{V}{T} \geq I_d \middle| \tilde{\Phi}_s\right\}\right], \tag{28}$$

$$= 1 - \mathbb{E}_{\tilde{\Phi}_s}\left[\mathbb{P}_{h_{sr}|\tilde{\Phi}_s}\left\{|h_{sr}|^2 \geq \frac{TI_r}{\mu_3} \middle| \tilde{\Phi}_s\right\} \mathbb{P}_{V|\tilde{\Phi}_s}\left\{\frac{V}{T} \geq I_d \middle| \tilde{\Phi}_s\right\}\right], \tag{29}$$

$$= 1 - \mathbb{E}_{\tilde{\Phi}_s}\left[e^{-\frac{TI_r}{\mu_3}} \bar{F}_V(TI_d)\right], \tag{30}$$

where $\bar{F}_V(\cdot)$ is the complementary cumulative distribution function (CCDF) of V and μ_3 is given by (24). Notice that in general V has the distribution of the sum of two independent exponential RVs with different means. The exception to this is when $\rho = 0$ and $||d-r|| = D$. In that case, the means of the exponential RVs are the same so V follows a Gamma distribution with 2 degrees of freedom. In precise terms, in Appendix D-A we show that the CCDF is:

$$\bar{F}_V(u) = \begin{cases} \frac{\mu_2 e^{-u/\mu_2} - \mu_1 e^{-u/\mu_1}}{\mu_2 - \mu_1} & \mu_1 \neq \mu_2 \\ (1 + u/\mu_1) e^{-u/\mu_1} & \mu_1 = \mu_2 \end{cases} \tag{31}$$

where μ_1 and μ_2 come from (22) and (23). Straightforward calculations show that $\mu_1 = \mu_2$ only if $\rho = 0$ and $||d-r|| = D$. Now, replacing that expression in (30) and using the definition of the Laplace transform (25) we obtain (21). To obtain (26) we replace (31) in (30) and use the fact that:

$$\frac{d\mathbb{E}_{\tilde{\Phi}_s}\left[e^{-(\omega_1 I_d + \omega_2 I_r)}\right]}{d\omega_1} = \mathbb{E}_{\tilde{\Phi}_s}\left[-I_d e^{-(\omega_1 I_d + \omega_2 I_r)}\right]. \tag{32}$$

■

Remark 3.1: Notice that the OP in this scenario is a split function because the fading RV V given by (27) has the same property. However, the expression of the OP when $\rho = 0$ and $||d-r|| = D$, given by (26), can be obtained by continuously extending the other expression

(21) at these points. Therefore, in what follows we shall focus our interest on (21) which fully characterizes the OP.

Now we consider the case of the simplified path loss function:

Corollary 3.1: For the simplified path loss function, when $||d-r|| \neq D$ or $\rho \neq 0$ the OP is:

$$\mathbb{P}_{\text{out,DF}}(R) = 1 - \frac{\mu_2}{\mu_2-\mu_1} e^{-\lambda_s\left[\delta\left(\mu_2^{-2/\alpha}+\mu_3^{-2/\alpha}\right)+f(T/\mu_2,T/\mu_3)\right]} + \frac{\mu_1}{\mu_2-\mu_1} e^{-\lambda_s\left[\delta\left(\mu_1^{-2/\alpha}+\mu_3^{-2/\alpha}\right)+f(T/\mu_1,T/\mu_3)\right]}, \quad (33)$$

where:

$$f(\omega_1,\omega_2) = \int_{\mathbb{R}^2} \frac{\omega_1\omega_2}{(\omega_1+||x-d||^\alpha)(\omega_2+||x-r||^\alpha)} dx, \quad (34)$$

$$C = \frac{2\pi}{\alpha}\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right), \quad (35)$$

$$\delta = CT^{2/\alpha}, \quad (36)$$

and $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ is the usual Gamma function.

Proof: Use Lemma A.1 in Appendix A to evaluate the Laplace transform in the previous Theorem. ■

Remark 3.2: The OP depends only on the absolute value of ρ and not on its phase. This is a consequence of the uniform phase of the Rayleigh fading coefficients.

Notice that $f(\omega_1,\omega_2)$ in (34) accounts for the statistical dependence between the interferences at two different locations. If in fact this two interferences were independent, then the joint Laplace transform would result in the product of their individual transforms and this cross-term would not appear. Unfortunately this term does not have a closed form and is very difficult to bound it tightly in a general setup for all (ω_1,ω_2). Therefore, although we can find numerically the value of $f(\omega_1,\omega_2)$ without difficulty, we cannot provide a closed form expression for direct computation. Since in general we assume that the relay is in the vicinity of the source, the union bound on the outage probability will be tight. That is:

$$\mathbb{P}_{\text{out,DF}}(R) \leq \inf_{\rho\in\mathbb{C},|\rho|\leq 1} \left(\mathbb{P}_\Theta\left\{\mathcal{A}(R,\rho)\right\} + \mathbb{P}_\Theta\left\{\mathcal{B}(R,\rho)\right\}\right), \quad (37)$$

will be a good approximation, since a close inspection of $\mathcal{A}(R,\rho)$ and $\mathcal{B}(R,\rho)$ shows that in this setting the event $\mathcal{B}(R,\rho)$ will be dominant and that $\mathcal{A}(R,\rho)$ will have a relatively small

probability of occurrence. The following Theorem deals with evaluating the union bound (37) and with the small node density regime:

Theorem 3.2 (Upper bounds on the OP for a single fixed relay and the SND regime): When only the source at the origin has a fixed relay and $||r - d|| \neq D$ or $\rho \neq 0$, considering the simplified path loss function, the OP can be upper bounded as:

$$\mathbb{P}_{\text{out,DF}}(R) \leq \inf_{\rho \in \mathbb{C}, |\rho| \leq 1} \left\{ 2 - e^{-\frac{\lambda_s \delta ||r||^2}{(1-|\rho|^2)^{2/\alpha}}} - \frac{\mu_2 e^{-\lambda_s \delta \mu_2^{-2/\alpha}} - \mu_1 e^{-\lambda_s \delta \mu_1^{-2/\alpha}}}{\mu_2 - \mu_1} \right\}, \tag{38}$$

with μ_1 and μ_2 given by (22) and (23). In addition in the small node density regime we can bound $\kappa(R)$ as:

$$\kappa(R) \leq \inf_{\rho \in \mathbb{C}, |\rho| \leq 1} \frac{\delta}{\pi} \left(\frac{\|r\|^2}{D^2 (1 - |\rho|^2)^{2/\alpha}} + \frac{1}{D^2} \frac{\mu_2^{1-\frac{2}{\alpha}} - \mu_1^{1-\frac{2}{\alpha}}}{\mu_2 - \mu_1} \right). \tag{39}$$

Proof: For the first part it is easy to show that:

$$\mathbb{P}_\Theta \left\{ \mathcal{A}(R, \rho) \right\} = 1 - \mathcal{L}_{I_r} \left(T/(1 - |\rho|^2) \right) = 1 - e^{-\frac{\lambda_s \delta ||r||^2}{(1-|\rho|^2)^{2/\alpha}}}, \tag{40}$$

which represents the OP of a DT from the source to the relay [3] with the exception of the correlation coefficient ρ which does not introduce any difficulty. In addition, the proof that:

$$\mathbb{P}_\Theta \left\{ \mathcal{B}(R, \rho) \right\} = 1 - \frac{\mu_2 \mathcal{L}_{I_d} (T/\mu_2) - \mu_1 \mathcal{L}_{I_d} (T/\mu_1)}{\mu_2 - \mu_1} \tag{41}$$

$$= 1 - \frac{\mu_2 e^{-\lambda_s \delta \mu_2^{-2/\alpha}} - \mu_1 e^{-\lambda_s \delta \mu_1^{-2/\alpha}}}{\mu_2 - \mu_1}, \tag{42}$$

follows along the same lines as Theorem 3.1. The Laplace transform has the same expression as (40) (it can be found by setting $\omega_2 = 0$ in (116) from lemma A.1 in Appendix A).

The proof of the second part follows directly from the fact that $e^{-u} = 1 - u + O(u^2)$ and the definition of $\kappa(R)$ (12). ■

Notice again, that the case where $||r - d|| = D$ and $\rho = 0$ can be treated via continuity arguments as mentioned above.

Now, it is important to adjust the value of ρ which determines the correlation between the source and relay signals. We do so in the SND regime.

Lemma 3.1 (Optimal ρ in the SND regime for the union bound): In the small node density regime, the right side of (37) is minimized by taking $\rho = 0$.

Proof: See appendix B-A. ■

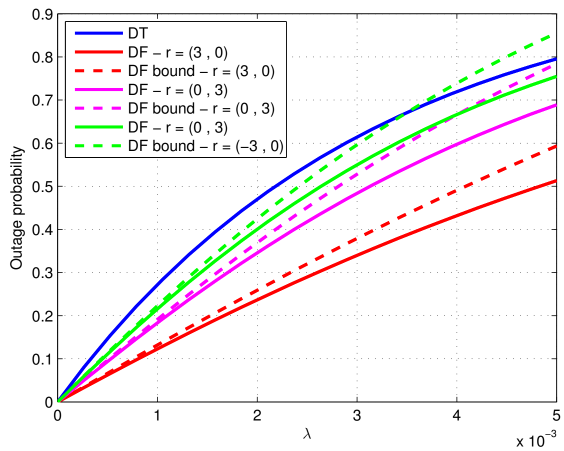


Fig. 2. OP when only the source at the origin has a relay located at r. Exact expressions for DF come from (33) while upper bounds come from (43). $d = (10, 0)$, $R = 0.5$, $\alpha = 4$.

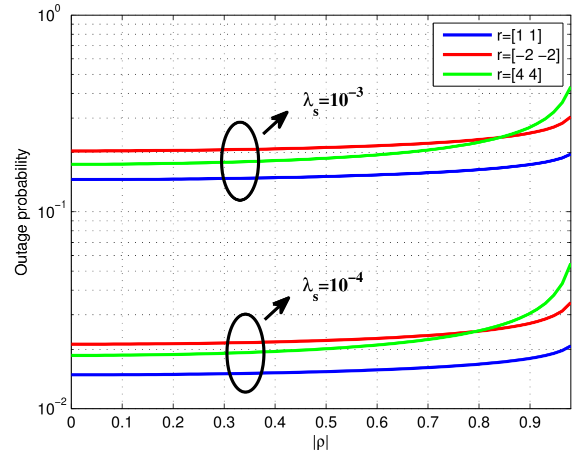


Fig. 3. Outage probability as a function of $|\rho|$ for various relay position and for $\lambda_s = 10^{-3}$ and $\lambda_s = 10^{-4}$. $d = (10\ 0)$, $\alpha = 4$ and $R = 0.5$.

Although the optimality of $\rho = 0$ is proved for the SND regime, we will see through numerical simulations that it is in fact true for practical values of the OP (21) .

Remark 3.3: As pointed out in [1] (see remark 42) and [7], using $\rho = 0$ simplifies the implementation of DF because symbol synchronization between the source and its corresponding relay, which could be difficult to obtain in ad hoc wireless network, is not strictly required. In what remains of this section we focus on the case $\rho = 0$.

Corollary 3.2: Using the simplified path loss function and when $||d - r|| \neq D$ the optimal OP ($\rho = 0$) can be upper bounded as:

$$\mathbb{P}_{\text{out,DF}}(R) \leq \left(1 - e^{-\lambda_s \delta ||r||^2}\right) + \left(1 - \frac{D^\alpha e^{-\lambda_s \delta ||r-d||^2} - ||r-d||^\alpha e^{-\lambda_s \delta D^2}}{D^\alpha - ||r-d||^\alpha}\right). \tag{43}$$

In the small node-density regime we have:

$$\kappa(R) \leq \frac{\delta D}{\pi} \left(\frac{||r||^2}{D^2} + ||r-d||^2 \frac{D^{\alpha-2} - ||r-d||^{\alpha-2}}{D^\alpha - ||r-d||^\alpha}\right). \tag{44}$$

Now we present a few figures to show the behavior of the expressions derived and to compare the performance of DF in this case. In Fig. 2 we can see the comparison of DF versus DT, both through the exact numerical evaluation of the OP using (33) and with the upper bounds given by (43) for different relay positions, taking $d = (10\ 0)$, $\alpha = 4$ and $R = 0.5$. We can see that these bounds are very accurate when the OP is small as proposed. In addition, for a fixed

source-relay distance the OP increases as the relay grows further away from the destination. This is because the probability of $\mathcal{B}(R,\rho)$ increases as this happens. In Fig. 3 we can observe how the variation of the true outage probability of equation (33) as a function of $|\rho|$ for various values of r, using the same parameters as in figure 2. Two sets of curves are presented. One for the case of $\lambda_s = 10^{-4}$, in which the OP is small, and the other for $\lambda_s = 10^{-3}$, in which the OP is larger. In both cases we see that $\rho =$ is the optimal choice.

B. Relay at a Random Position

In this section we consider that only the node at the origin has a relay with probability one, and that this relay is chosen as the NN of the source at the origin. Thus, the expressions for the OP found in the previous section can be interpreted as being conditioned on the position of the relay so now for a random relay located at r the OP can be expressed as[5]:

$$\mathbb{P}_{\text{out,DF}}(R) = \inf_{\rho\in\mathbb{C},|\rho|\leq 1} \mathbb{E}_r\left[\mathbb{P}_{\Theta|r}\left\{\mathcal{A}(\rho,R)\cup\mathcal{B}(\rho,R)|r\right\}\right]. \quad (45)$$

In the previous section the OP for a fixed known location of the relay was calculated and it was discussed that for the small node-density regime the optimal correlation coefficient between the symbols of the source and the relay is in fact $\rho = 0$. Assuming the same operating condition, the probability inside the expectation should be minimized for all values of r when $\rho = 0$. Therefore in this scenario we can write:

$$\mathbb{P}_{\text{out,DF}}(R) = \mathbb{E}_r\left[\mathbb{P}_{\Theta|r}\left\{\mathcal{A}(R)\cup\mathcal{B}(R)|r\right\}\right], \quad (46)$$

where we define $\mathcal{A}(R)\equiv\mathcal{A}(R,0)$ and $\mathcal{B}(R)\equiv\mathcal{B}(R,0)$. It is clear that the OP for the simplified path loss function with $||r-d||\neq D$ (and $\rho = 0$) can be calculated by averaging (33) from corollary 3.1 with respect to the relay position r:

$$\mathbb{P}_{\text{out,DF}}(R) = 1-\mathbb{E}_r\left[\frac{D^\alpha}{D^\alpha-||r-d||^\alpha}e^{-\lambda_s\left[\delta(||r||^2+||r-d|^2)-f(T||r-d||^\alpha,T||r||^\alpha)\right]} - \frac{||r-d||^\alpha}{D^\alpha-||r-d||^\alpha}e^{-\lambda_s\left[\delta(||r||^2+D^2)-f(TD^\alpha,T||r||^\alpha)\right]}\right]. \quad (47)$$

The values of r for which $||r-d|| = D$ do not affect the value of the expectation since at those points (47) has an avoidable singularity. From the last section we know that the function

[5] In this case the randomness in the model is given by $\Theta = \left\{\tilde{\Phi}_s, h_{sr}, h_{rd}, h_{sd}, r\right\}$ where r is now random, and we also remove the marks in $\tilde{\Phi}_s$ related to the relays of the other clusters.

$f(\cdot,\cdot)$ cannot be computed in closed form. Moreover, it is very difficult to upper and lower bound accurately in a general scenario. This precludes in turn a closed form computation of the expectation with respect to r as it is required in this scenario. Notice also that each term inside the expectation does not have finite expectation separately so the expectation in (47) should be evaluated jointly. For this reason, to derive closed form expressions we have to find a suitable upper bound which can be averaged with respect to r. In the previous section we showed that in the SND regime the union bound is tight, and therefore it will also be useful in this scenario. Therefore we upper bound the OP as:

$$\mathbb{P}_{\text{out,DF}}(R) \leq \mathbb{E}_r\left[\mathbb{P}_{\Theta|r}\left\{\mathcal{A}(R)|r\right\} + \mathbb{P}_{\Theta|r}\left\{\mathcal{B}(R)|r\right\}\right]. \tag{48}$$

The next lemma deals with the nearest neighbor distribution of a HPPP which we require for averaging:

Lemma 3.2: The position of the node closest to the origin of an homogeneous Poisson point process of intensity λ_{un} is a two-dimensional Gaussian r.v. with zero mean, independent components and variance $(2\lambda_{in}\pi)^{-1}$, that is, its density is:

$$f_{in}(r) = \frac{1}{2\pi\sigma_{in}^2} e^{-\frac{||r||^2}{2\sigma_{in}^2}} = \lambda_{in} e^{-\lambda_{in}\pi||r||^2}. \tag{49}$$

Proof: The proof follows from the fact that the distance from the origin to the closest point in the Poisson process is distributed as a Rayleigh RV. Also, from the homogeneity of the point process, it follows that the angle of vector r should be a uniform RV in $[0, 2\pi)$, independent of its norm. ∎

The expression inside the expectation in (48) is in fact (43), which we now want to average with respect to r. The first term is due to $\mathcal{A}(R)|r$ and its average has a closed form expression. The second term is due to $\mathcal{B}(R)|r$ and cannot be averaged in closed form. Moreover, it has an avoidable singularity at $||r-d|| = D$ which further complicates matters. Therefore we propose to find a simpler upper bound on the probability of this event. We restrict ourselves to the case $\alpha \geq 4$.

Lemma 3.3: When $\alpha \geq 4$, $\mathbb{P}_\Theta\left\{\mathcal{B}(R)|r\right\}$ can be upper bounded as:

$$\mathbb{P}_\Theta\left\{\mathcal{B}(R)|r\right\} \leq 1 - \left(N - M||r-d||\right) e^{-\lambda_s \delta D^2}, \tag{50}$$

where:

$$N = 1 + \lambda_s \delta D^2 + \frac{2(\lambda_s \delta D^2)^2}{\alpha} \tag{51}$$

$$M = \lambda_s \delta D \left(1 - \frac{2}{\alpha}(1 - \lambda_s \delta D^2)\right) \tag{52}$$

for any set of values (λ_s, T, D).

Proof: See appendix B-B. ■

The usefulness of the previous lemma relies on the fact that the bound is tight when $||r-d|| \approx D$ (for example near the origin) and that it is linear in $||r-d||$ so it can be averaged in closed form. Notice that this bound becomes negative if $||r-d||$ is too large; however it is easy to show that if $\lambda_s \delta D^2 \leq 1$ (even well out of the SND regime) this bound will be positive at least when $||r-d|| \leq 2D$. Since the relay will be close to the source, the contribution of the negative parts will be negligible after averaging. With this bound we can upper bound the OP in the small outage regime:

Theorem 3.3 (Upper bounds on the OP for a single random relay): When $\alpha \geq 4$ the OP for the DF protocol when only the source at the origin has a relay following the NN distribution and for the simplified path loss function, can be upper bounded as:

$$\mathbb{P}_{\text{out,DF}}(R) \leq 1 - (N - M\mathbb{E}_r\left[||r-d||\right]) e^{-\lambda_s \delta D^2} + \frac{\lambda_s \delta}{\pi \lambda_{in} + \lambda_s \delta}, \tag{53}$$

where N and M come from (51) and (52). In addition in the SND regime, we have:

$$\kappa(R) \leq \frac{\delta}{\pi}\left[\frac{2\sigma_{in}^2}{D^2} + \left(1 - \frac{2}{\alpha}\right)\frac{\mathbb{E}_r\left[||r-d||\right]}{D}\right]. \tag{54}$$

For the expectation we can use:

$$\mathbb{E}_r\left[||r-d||\right] = \sigma_{in} Q_{2,0}(D/\sigma_{in}, 0), \tag{55}$$

$$\mathbb{E}_r\left[||r-d||\right] \leq D + \gamma\left(\frac{\sigma_{in}}{D}\right)\sigma_{in}, \tag{56}$$

where $Q_{2,0}$ is the $(2,0)$ Nuttall Q-function [31]. In particular:

$$Q_{2,0}(s,0) = \sqrt{\frac{\pi}{8}} e^{-\frac{s^2}{4}} \left(\left(s^2+2\right) I_0\left(\frac{s^2}{4}\right) + s^2 I_1\left(\frac{s^2}{4}\right)\right),$$

where I_0 and I_1 are the modified Bessel functions of the first kind of orders 0 and 1. Similarly $\gamma(s)$ is defined by:

$$\gamma(s) = \sqrt{\frac{\pi}{2}}\left[1 + \left(\frac{4}{\pi} - 2\right)\text{erf}\left(\frac{1}{\sqrt{2}s}\right)\right], \tag{57}$$

where erf$(\cdot)$ is the standard error function.

Proof: The first two terms in (53) come from using lemma 3.3 to bound $\mathbb{P}_{\Theta|r}\{\mathcal{B}(R)|r\}$, which is the second term of (43), and averaging with respect to r. The third term in (53) follows from averaging $\mathbb{P}_{\Theta|r}\{\mathcal{A}(R)|r\}$, which is the first term of (43), with respect to the relay distribution (49). The SND regime result once more follows from the fact that $e^{-u} = 1-u+O(u^2)$ and the definition of $\kappa(R)$ (12). Finally for the proof of the results on the expectation of $||r-d||$ see appendix B-C. ∎

Remark 3.4: (55) will provide tighter expressions than (56). However, (56) will allow further analysis in the next section when we shall include the possibility of a helping relay to all source nodes in the network. $\gamma(s)$ is monotonically increasing so its minimum value is attained as $s \to 0$ when $\gamma(s) \to \left(\frac{4}{\pi} - 1\right) \approx 0.3425$. In addition, if $s \leq 1/3$ it is almost constant. Notice that if an upper bound is determined for s then $\gamma(s)$ can be upper bounded by a constant so (56) can simplified to a linear function of σ_{in}.

To finish this section we present a few figures to study the performance of the bounds defined in Theorem (3.3). In figure 4 we can see the OP for a random relay for different values of σ_{in} obtained both through numerical simulation of (47) and by using (55) in (53) against DT. In all cases we take $\alpha = 4$, $R = 0.5$ and $d = (10, 0)$. We observe that the bounds derived predict the OP in the small outage regime very accurately. It is clear that as σ_{in} grows, that is, as the NN distribution becomes less concentrated around the source, the probability of finding the relay close to the source decreases and on average the performance of DF will decrease. In figure 5 we plot the maximum achievable rates attainable through DF relative to the rate of DT for an OP constraint, as a function of the NN dispersion. The DF rates are obtained by using (55) in (53); since this is an upper bound on the OP, the plotted gains are actually a (tight) lower bound of the actual gains. It is clear that as the spread of the NN decreases (σ_{in}/D decreases) the average gains become larger.

IV. Multiple sources and relays

In the previous section we considered the case in which only the cluster at the origin has the possibility of cooperating. This was done to isolate and analyze how a situation of strong interference could affect the performance of the DF scheme of a given user in the network. Although this analysis is important in itself it only provides a local understanding of cooperation

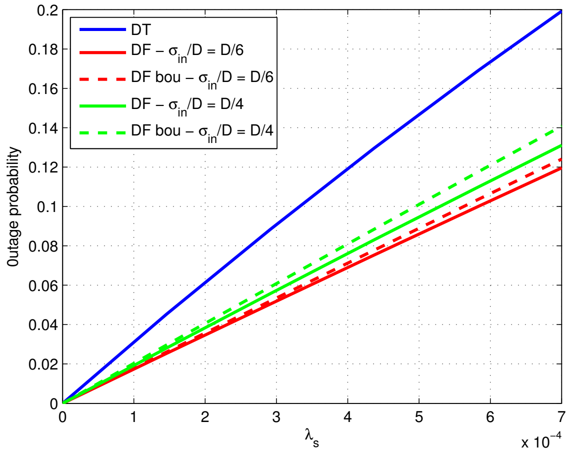


Fig. 4. OP when only the source at the origin has random relay. The exact OP for DF comes from numerically evaluating (47), while bounds come from using (55) in (53). $d = (10, 0)$, $R = 0.5$, $\alpha = 4$.

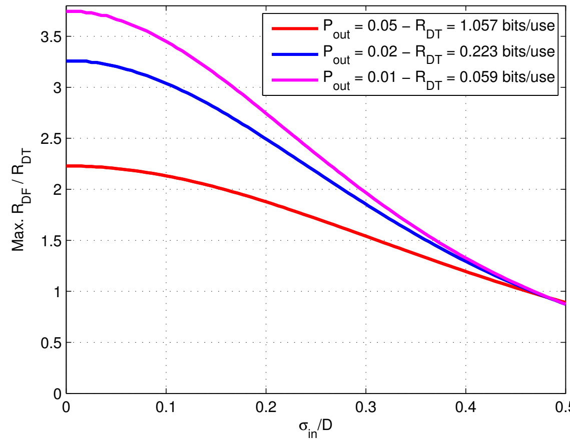


Fig. 5. Maximum DF rate relative to the same rate of DT for a fixed OP. DF rates are obtained by using (55) in (53). $\lambda = 10^{-4}$. $d = (10, 0)$, $\alpha = 4$.

since only one user is allowed to use a relay. This is similar to the standard AWGN fading relay channel. In this section we shall consider the more interesting situation in which relays are available for all source nodes. As we shall see, many of the tools developed in the previous section will aid us in what follows. We start by discussing a useful approximation which will lead to a mathematically tractable problem.

A. A Useful Approximation

Even with the simple model presented in section II, the problem at hand is mathematically difficult to attack. The expression of the OP for the transmission scheme detailed in section II is given by equation (18). In more precise terms, that equation can be expressed as:

$$\mathbb{P}_{\text{out,mix}}(R) = \inf_{0 \le p_r \le 1} \mathbb{P}_{\text{out,mix}}(R, p_r), \tag{58}$$

where $\mathbb{P}_{\text{out,mix}}(R, p_r)$ denotes the outage probability of the scheme for a fixed value of p_r. Using the tools presented in appendix A it can be shown that $\mathbb{P}_{\text{out,mix}}(R, p_r)$ can be written as:

$$\mathbb{P}_{\text{out,mix}}(R, p_r) = \inf_{\rho \in \mathbb{C}, |\rho| \le 1} p_r \mathbb{E}_r\left[1 - \frac{\mu_2}{\mu_2 - \mu_1}\mathcal{L}_{I_d, I_r}(T/\mu_2, T/\mu_3) + \frac{\mu_1}{\mu_2 - \mu_1}\mathcal{L}_{I_d, I_r}(T/\mu_1, T/\mu_3)\right] \\ + (1 - p_r)\mathcal{L}_{I_d}\left(T/l_{sd}\right), \tag{59}$$

where μ_1, μ_2 and μ_3 are defined in (22), (23) and (24) respectively. The interferences I_r and I_d are given by (5) and (6) and the two-dimensional Laplace transform $\mathcal{L}_{I_d,I_r}(\omega_1,\omega_2)$ are given by

$$\mathcal{L}_{I_d,I_r}(\omega_1,\omega_2) = \exp\left\{-\lambda_s p_r t(\omega_1,\omega_2,r,d,\rho)\right\} \exp\left\{-\lambda_s(1-p_r)\left[C(\omega_1^{2/\alpha}+\omega_2^{2/\alpha})+f(\omega_1,\omega_2)\right]\right\}, \tag{60}$$

where $f(\omega_1,\omega_2)$ is given by (34) and $t(\omega_1,\omega_2,r,\rho)$ is given by:

$$t(\omega_1,\omega_2,r,d,\rho) = \int_{\mathbb{R}^2} \mathbb{E}_k\left[1 - s\left(\omega_1,x,k,d,\rho\right) s\left(\omega_2,x,k,r,\rho\right)\right] dx. \tag{61}$$

The expectation is with respect to the distribution[6] of k and where $s(\omega,x,k,d,\rho)$ is given by:

$$s(\omega,x,k,d,\rho) = \frac{1}{1+\omega\|x-d\|^{-\alpha}+\omega\|k-d\|^{-\alpha}+(1-|\rho|^2)\omega^2\|x-d\|^{-\alpha}\|k-d\|^{-\alpha}}. \tag{62}$$

For $s(\omega,x,k,r,\rho)$ a similar expression holds interchanging d with r. Notice the complexity of the true OP. It is clear that expression (59) is not amenable for mathematical analysis. Moreover, obtaining (59) numerically implies significant computational resources because of the need of performing integration routines in high dimensional spaces (for example $\mathbb{R}^6$, if the relay are distributed with the NN distribution). For this reason, we shall make another simplifying assumption. The variances of the interferences (conditioned on Φ_s) in the relay and the destination associated with the source at the origin are given by (5) and (6). These expressions prove to be very difficult to work with, specially if we are interested in simple final closed form results, which could lead to mathematically tractable analyzes. Besides the statistical dependence between I_d and I_r, the problem is given by the different path loss attenuations, that for example, the destination or the relay of the source at the origin, see from a given active node and its associated relay. This difference in path loss attenuation lead to integrals on $\mathbb{R}^2$ or $\mathbb{R}^4$ which cannot solved in closed form or be tightly bounded (from below and above). As we are interested in simple expressions which could shed light on the problem at hand, we shall avoid this problem by considering a simplified path loss attenuation:

$$\tilde{I}_r = \sum_{i:x_i\in\Phi_s} \frac{|h_{x_i r}|^2 + \varepsilon_{x_i}\left(|h_{k_i r}|^2 + 2\Re\left\{h_{x_i r}h_{k_i r}^*\rho\right\}\right)}{\|x_i+\tau k_i - r\|^\alpha}, \tag{63}$$

$$\tilde{I}_d = \sum_{i:x_i\in\Phi_s} \frac{|h_{x_i d}|^2 + \varepsilon_{x_i}\left(|h_{k_i d}|^2 + 2\Re\left\{h_{x_i d}h_{k_i d}^*\rho\right\}\right)}{\|x_i+\tau k_i - d\|^\alpha}, \tag{64}$$

[6] According to our model in section II k is distributed as a degenerate RV in $\mathbb{R}^2$ or with the NN from Φ_{in} distribution.

with $\tau \in [0,1]$. The main difference between these expressions and the ones in (5) and (6) is given by the fact that we have approximated the two path losses appearing in the interferences by a single one. For the case of the destination of the source at the origin this becomes:

$$\frac{1}{\|x_i - d\|^{\alpha}} \approx \frac{1}{\|x_i + \tau k_i - d\|^{\alpha}}, \qquad \frac{1}{\|x_i + k_i - d\|^{\alpha}} \approx \frac{1}{\|x_i + \tau k_i - d\|^{\alpha}}. \tag{65}$$

Similar expressions apply for the relay. Consider the term $\|x_i + \tau k_i - d\|^{-\alpha}$, for what it is easy to show that:

$$\frac{1}{\|x_i + \tau k_i - d\|^{\alpha}} = \frac{1}{\|x_i - d\|^{\alpha}} + O\left(\alpha \frac{\|k_i\|}{\|x_i - d\|}\right). \tag{66}$$

So, in the case $\|x_i - d\| \gg \alpha \|k_i\|$ the approximation should be good. Of course, this cannot be guaranteed in an exact way, because of the random locations of the nodes x_i. However, if λ_s is small enough, this can be assured with high probability. It is easy to show that:

$$\mathbb{P}_{\Theta}\left\{\|x - d\| < \alpha \|k\|\|\right\} = 1 - \mathbb{E}_k\left[e^{-\lambda_s \pi \alpha \|k\|^2}\right], \tag{67}$$

where the latter expectation is with respect to a generic mark k. Notice that under the assumptions in section II, the distribution of k (as a fixed vector or as the NN distribution of Φ_{in}) is independent of the particular x at which is associated, that is, the marked HPPP assumed for the model is stationary [13]. When the relay position is fixed and if $\lambda_s \|k\|^2$ is small enough, the probability of having a network configuration in which the approximation was not good should be small. In the case when k is distributed with the nearest neighbor distribution of Φ_{in},

$$\mathbb{P}_{\Theta}\left\{\|x - d\| < \alpha \|k\|\|\right\} = \frac{\alpha \lambda_s}{\lambda_{in} + \alpha \lambda_s}, \tag{68}$$

and the same would be true depending on λ_s and λ_{in}. Typically, in order to have useful levels of OP performance, the number of active nodes per unit area, λ_s, should smaller than the number of inactive nodes per unit of area λ_{in}. In that sense $\mathbb{P}_{\Theta}\left\{\|x - d\| < \alpha \|k\|\|\right\}$ should be small. We should mention, that the above reasoning is meant to be an intuitive argument and not a formal proof to validate (63) and (64). However, the numerical results will show that this is indeed a good approximation. We will use that approximation below in order to analyze the OP of the typical cluster and its dependence with p_r. First we will analyze the problem when the relay is collocated with the source and then when it is not collocated but it is at a fixed position. Finally we shall treat the problem when the relay is distributed around each source with the NN distribution of Φ_{in}. In the following we shall also impose that $\rho = 0$ and although we will be

working with (63) and (64) we will still use the symbols I_r and I_d just to simplify the notation.

Remark 4.1: We should note that the choice of $\rho = 0$ will not improve necessarily the global performance of the network. This is because if $\rho \neq 0$ the Gaussian interference generated by a source-relay pair will be correlated. For another node subject to this interference that correlation could be beneficial. On the other hand, from a local point of view, $\rho = 0$ could improve the OP performance of the DF scheme (see section III) so there is another interesting balance between local and global network performance. In this work, however, we will not consider this problem.

B. Relays Collocated With their Sources

We start by analyzing the case in which the relay is collocated with the source [8] in all the clusters, because this setup is mathematically tractable and it will give us some interesting insights on the more general problem. That is, we assume that the source-relay channels in all the clusters are not subject to amplitude fading and only to a fixed common gain given by $g \in \mathbb{C}$ (with the possible presence of phase fading) known to the relays. The relays are not impaired by the interference I_r and only by complex, circular Gaussian noise with unit variance. This could model the situation in which the relays have out-of-band reception and in-band transmission [32], [33], that is, the source-relay links are orthogonal to the source-destination and relay-destination links, which could happen for example, if the source-relay links are wired. The relays are also at relative distance r ($k_i = r \ \forall i$) from their corresponding source nodes, in such a way that the approximation (64) at the destinations is valid (obviously approximation (63) is not needed in this case), and their channels to their corresponding destinations are subject to fading and path loss. As usual the sources and relays radios transmit with unit power. The sources should also invest some power in the transmission to the relays through their corresponding links, which is already considered in the channel gain g.

It is clear that with this simplified model we still capture the effect we are interested in. Although the relays reception is not impaired by the network interference, in order to obtain local cooperative gains through DF, the relays have to transmit (in a wireless fashion) introducing interference in the network and degrading the global performance.

Let us consider the Bernoulli RV ε_0 indicating that the source at the origin uses a relay. To find the OP we can condition on this RV; when the node at the origin does not use a relay then

the OP is the same as DT, and when it does, it is the OP of the DF protocol. So we can write the outage probability (with $\rho = 0$) as[7]:

$$\mathbb{P}_{\text{out,mix}}(R, p_r) = \mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{A}(R) \cup \mathcal{B}(R)|\varepsilon_0 = 1\right\}\mathbb{P}_{\varepsilon_0}(\varepsilon_0 = 1) + \mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{A}_{DT}(R)|\varepsilon_0 = 0\right\}\mathbb{P}_{\varepsilon_0}(\varepsilon_0 = 0), \tag{69}$$

where we used $\mathcal{A}(R) \equiv \mathcal{A}(R, 0)$ and $\mathcal{B}(R) \equiv \mathcal{B}(R, 0)$. The event $\mathcal{A}(R)$ has probability one or zero depending on $\log(1 + |g|^2) \geq R$ or $\log(1 + |g|^2) < R$. As we are interested in exploiting the relay if it is present, we restrict ourselves to the case $\log(1 + |g|^2) \geq R$. In this way, the relay, if present, is always able to decode the source message. Equation (69) becomes:

$$\mathbb{P}_{\text{out,mix}}(R, p_r) = \mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{A}_{DT}(R)|\varepsilon_0 = 0\right\}\mathbb{P}_{\varepsilon_0}(\varepsilon_0 = 0) + \mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{B}(R)|\varepsilon_0 = 1\right\}\mathbb{P}_{\varepsilon_0}(\varepsilon_0 = 1). \tag{70}$$

We have the following Theorem:

Theorem 4.1 (OP for collocated relays in all clusters): The OP when the relays are collocated with their sources in all clusters (eq. (70)) is:

$$\begin{aligned}\mathbb{P}_{\text{out,mix}}(R, p_r) = p_r\left(1 - \mathcal{L}_{I_d}\left(T/l_{sd}\right)\right) + (1 - p_r)\\ \times\left(1 - \frac{D^\alpha \mathcal{L}_{I_d}\left(T/l_{rd}\right)}{D^\alpha - \|r - d\|^\alpha} + \frac{\|r - d\|^\alpha \mathcal{L}_{I_d}\left(T/l_{sd}\right)}{D^\alpha - \|r - d\|^\alpha}\right),\end{aligned} \tag{71}$$

where:

$$\mathcal{L}_{I_d}(\omega_1) = \exp\left\{-\lambda_s C \omega_1^{2/\alpha}\left(1 + \frac{2p_r}{\alpha}\right)\right\}, \tag{72}$$

and C is defined as in (35) and δ is given by (36).

Proof: The proof of the expressions in terms of the Laplace transforms can be obtained along the same lines as Theorem 3.1. The expression of the Laplace transform can be found in lemma A.2 in appendix A. ■

Notice that the p_r appearing in the exponentials of (71) accounts for the additional interference introduced by the other active relays in the network. On the other hand, the values of p_r and $1-p_r$ that weigh these exponentials account for the possible local improvements due to cooperation in the typical cluster. It is easy to verify that when $p_r = 0$ (there are no active relays with probability one), $\mathbb{P}_{\text{out,mix}}(R, 0) = \mathbb{P}_{\text{out,DT}}(R)$.

[7] We are making an abuse of notation in saying that (69) is the true outage probability of (59). This not the case basically for the path loss approximation mentioned in subsection IV-A. However, as (69) will be an excellent approximation to the true outage probability and in order to avoid introducing a new symbol we will use the same notation for this approximation.

The following lemma deals with the optimal relay activation probability:

Theorem 4.2 (Concavity of p_r for collocated relays): If the following inequality holds true:

$$\lambda_s \delta \max\left\{\|r-d\|^2, D^2\right\} \leq 0.38, \tag{73}$$

then $\mathbb{P}_{\text{out}}(R, p_r)$ (eq. (70)) is concave function in p_r.

Proof: See appendix C-A. ■

We have then the following corollary:

Corollary 4.1 (Optimal p_r for collocated relays): For the collocated relay case introduced above, if the destination and relay positions satisfy:

$$\lambda_s \delta \max\left\{\|r-d\|^2, D^2\right\} \leq 0.38, \tag{74}$$

we have that $\mathbb{P}_{\text{out}}(R, p_r)$ attains its minimum value when $p_r = 0$ or $p_r = 1$.

Proof: It follows from the fact that given a concave function $h(x)$ in a bounded and closed interval $[x_1, x_2]$, its minimum is attained at x_1 or x_2 (see Theorem 32.1 in [34]). ■

This means that under condition (74) the best OP performance for any cluster in the network can be attained when all or none of the sources decide to use their associated relays. In one case all the clusters will be using DF and in the other case all of them will be using DT. This is a somewhat surprising result in the sense that in terms of the OP the best performance can be obtained either by full cooperation or by not cooperating at all. There is no "optimal" density of used relays in the network or optimal mixed behavior in the sense that some clusters would enjoy the advantages of cooperation while others use DT in order to balance the generated interference. The conditions under which $p_r = 1$ or $p_r = 0$ would be the preferred choice depend on several network parameters (α, d, r, λ_s, R, etc). To see that, in the following subsection we treat the special case when $r = 0$, that is when the relays are physically collocated with the sources.

Remark 4.2: The result of lemma 4.2 is only a sufficient condition for $\mathbb{P}_{\text{out,mix}}(R, p_r)$ to be concave. It is not a necessary condition. In fact, our numerical results suggest the condition

$$\lambda_s \delta \max\left\{\|r-d\|^2, D^2\right\} \leq \frac{1}{2}, \tag{75}$$

is also sufficient for the concavity of $\mathbb{P}_{\text{out}}(R, p_r)$. Notice that (75) covers almost all the important conditions of operation. Consider that when $\lambda_s \delta D^2 = 0.5$, the outage probability for DT is $\mathbb{P}_{\text{out,DT}}(R) \approx 0.4$ which already corresponds to a very poor network performance.

C. Relays Physically Collocated With their Sources

In order to apply the results derived above we will consider the ideal situation in which the relays are physically collocated with their sources. It is easy to see that this corresponds to the case of a MISO system (2 transmitters-1 receiver), and it achieves the second term in the DF rate expression (14). The same situation happened in the case treated in the last subsection. The difference is that both antennas did not present the same average gain due to the different path losses they were subject to. In this case, since both of them are located in the same position, they have the same average gain. However, in the cases treated before and in this one, there is a fundamental difference with respect to the usual MISO case: in some sources the second antenna might not be used. In fact, it will be used with probability p_r. Therefore, the balance between the improved local performance and interference generation is considered through the use of the second transmitting antenna. The OP for this case can be exactly computed along the same lines of Theorem 4.1 and for that reason we skip the proof of the following result:

Theorem 4.3 (OP for physically collocated relays): The outage probability $\mathbb{P}_{\text{out,mix}}(R, p_r)$ for the case in which the relays are physically collocated with the sources is:

$$\mathbb{P}_{\text{out,mix}}(R, p_r) = 1 - \left[1 + \frac{2\lambda_s \delta D^2 p_r}{\alpha} + \frac{4\lambda_s \delta D^2 p_r^2}{\alpha^2}\right] \exp\left\{-\lambda_s \delta D^2 \left(1 + \frac{2p_r}{\alpha}\right)\right\}. \qquad (76)$$

The following lemma identifies the exact optimal values of p_r as a function of the other parameters of the network:

Theorem 4.4 (Optimal p_r for physically collocated relays): The optimal value p_r^{opt} for the case that the relay is physically collocated with its source is:

$$p_r^{\text{opt}} = \begin{cases} 1 & 0 < \eta \leq \frac{\alpha}{1+\frac{\alpha}{2}} \\ \alpha\left(\frac{1}{\eta} - \frac{1}{2}\right) & \frac{\alpha}{1+\frac{\alpha}{2}} < \eta \leq 2 \\ 0 & \eta > 2 \end{cases} \qquad (77)$$

where $\eta \equiv \lambda_s \delta D^2$. In the SND regime, where the optimal p_r^2 is one:

$$\kappa(R) \leq \frac{\delta}{\pi}\left(1 - \frac{4}{\alpha^2}\right). \qquad (78)$$

Proof: The proof follows standard procedures of function optimization and for that reason is omitted. ■

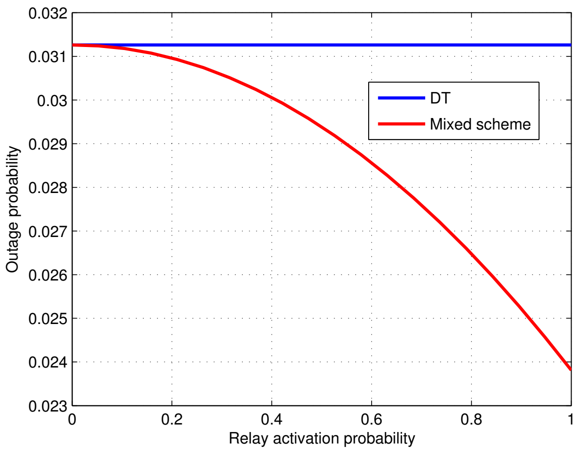


Fig. 6. $\mathbb{P}_{\text{out,mix}}(R, p_r)$ and $\mathbb{P}_{\text{out,DT}}(R)$ as function of p_r. $D = 10$, $\lambda_s = 10^{-4}$, $\alpha = 4$, $R = 0.5$.

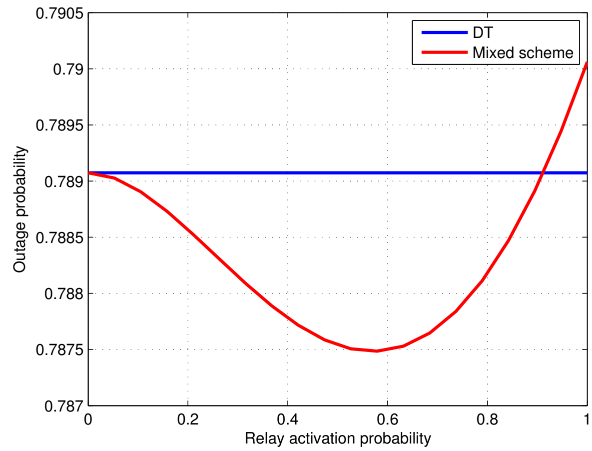


Fig. 7. $\mathbb{P}_{\text{out,mix}}(R, p_r)$ and $\mathbb{P}_{\text{out,DT}}(R)$ as function of p_r. $D = 70$, $\lambda_s = 10^{-4}$, $\alpha = 4$, $R = 0.5$.

Notice that in this ideal case the dependence of the optimal p_r on the network parameters is condensed on α and η. As $\mathbb{P}_{\text{out,DT}}(R) = 1 - e^{-\eta}$ all the above results can be put as functions that depend solely on the path loss exponent and the performance of DT with the actual network parameters.

In Fig. 6, 7 and 8, we see the outage probability of the mixed scheme and DT versus p_r. It is clear that DT does not depend on p_r. In Fig. 6 we see the case where the network is operating in $0 < \eta \leq \frac{\alpha}{1+\frac{\alpha}{2}}$ and for that reason $p_r = 1$ is optimal. In Fig. 7 the operating point is such that $\frac{\alpha}{1+\frac{\alpha}{2}} < \eta \leq 2$. In this case the optimal p_r is not 0 or 1 (we see that $\mathbb{P}_{\text{out,mix}}(R, p_r)$ is not concave). Finally, in Fig. 3 we have $\eta > 2$ and DT is optimal network-wide. In this case, we also have that $\mathbb{P}_{\text{out,mix}}(R, p_r)$ is not concave. So, in this particular case of physically collocated relays, in the concavity region obtained in Theorem 4.2, $p_r = 1$ is optimal. In other cases this is not true as we shall see later. It is important to mention that the cases in Fig. 7, and 8 occurs when the outage probability performance is far from practical and typical values. That is, from a practical standpoint, $p_r = 1$ is optimal.

In Fig. 9 we see how the optimal p_r behaves as a function of $\mathbb{P}_{\text{out,DT}}(R)$ for various values of α. We can observe that, for a large range the optimal probability is equal to 1. We also see that, for all values of α, the region where 1 or 0 is not the optimal value of p_r is sufficiently small. Finally, independently of α, when $\mathbb{P}_{\text{out,DT}}(R) > 0.865$ the optimal value is zero. We see there is

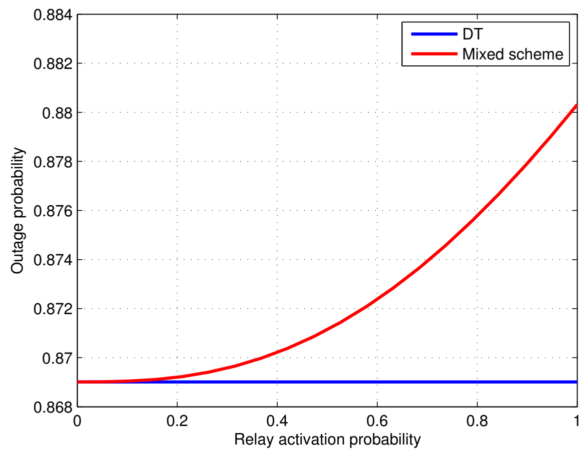


Fig. 8. $\mathbb{P}_{\text{out,mix}}(R, p_r)$ and $\mathbb{P}_{\text{out,DT}}(R)$ as function of p_r. $D = 80$, $\lambda_s = 10^{-4}$, $\alpha = 4$, $R = 0.5$.

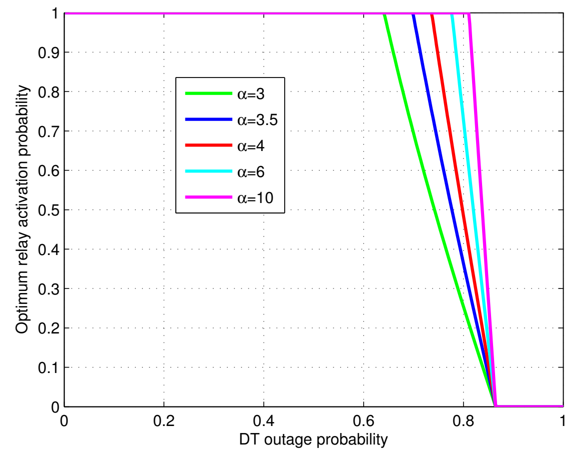


Fig. 9. Optimal relay activation probability p_r as a function of $\mathbb{P}_{\text{out,DT}}(R)$. $D = 10$, $\lambda_s = 10^{-4}$, $R = 0.5$.

an almost binary behavior, but this occurs far from the typical small node-density regime. This also shows what was mentioned above: from a practical standpoint, for this ideal case, $p_r = 1$ is optimal and cooperation is beneficial globally.

D. Relays at a Fixed Position but not Collocated

We shall consider now the more important case where the relay is at a fixed relative position with respect to its source for all the network clusters but it is not collocated. That is, the source-relay link, when it is used, is subject to fading, path loss and network interference. The outage probability is given by (69) (rewritten below for easy reference):

$$\mathbb{P}_{\text{out,mix}}(R, p_r) = \mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{A}(R) \cup \mathcal{B}(R) | \varepsilon_0 = 1\right\} \mathbb{P}_{\varepsilon_0}(\varepsilon_0 = 1) + \mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{A}_{DT}(R) | \varepsilon_0 = 0\right\} \mathbb{P}_{\varepsilon_0}(\varepsilon_0 = 0). \tag{79}$$

Again, we shall use the union bound in order to provide some tractable expression. The exact expression of $\mathbb{P}_{\text{out,mix}}(R, p_r)$ is given by (59). The use of that expression will complicate the analysis without providing any significant improvement in the accuracy. As mentioned in section III, if the relay is positioned in the vicinity of its source, the union bound should be a reasonable and accurate expression, because the event $\mathcal{B}(R)$ will be dominant with respect to $\mathcal{A}(R)$. Remember also that the assumption of the relays located in the vicinity of its sources was

also considered in order to use the approximation in the path loss attenuation in the subsection IV-A. In this way we can write:

$$\mathbb{P}_{\text{out,mix}}(R,p_r) \leq \left(\mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{A}(R)|\varepsilon_0=1\right\} + \mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{B}(R)|\varepsilon_0=1\right\}\right)\mathbb{P}(\varepsilon_0=1) + \mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{A}_{DT}(R)|\varepsilon_0=0\right\}\mathbb{P}(\varepsilon_0=0). \quad (80)$$

We have the following Theorem:

Theorem 4.5 (OP for fixed non-collocated relays): For the situation in which the relays are fixed at relative position r with respect to the sources, and subject to fading and network interference we have:

$$\mathbb{P}_{\text{out,mix}}(R,p_r) \leq (1-p_r)\left[1-\mathcal{L}_{I_d}\left(T/l_{sd}\right)\right] + p_r\left[1 - \frac{D^\alpha \mathcal{L}_{I_d}\left(T/l_{rd}\right)}{D^\alpha - \|r-d\|^\alpha} + \frac{||r-d||^\alpha \mathcal{L}_{I_d}\left(T/l_{sd}\right)}{D^\alpha - \|r-d\|^\alpha}\right] + p_r\left[1-\mathcal{L}_{I_r}\left(T/l_{sr}\right)\right], \quad (81)$$

where:

$$\mathcal{L}_{I_d}(\omega_1) = \exp\left\{-\lambda_s C \omega_1^{2/\alpha}\left(1+\frac{2p_r}{\alpha}\right)\right\}, \quad (82)$$

and δ is given by (36) and C is defined as in (35).

Proof: The proof follows the lines of Theorem 4.1. The new term $\mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{A}(R)|\varepsilon_0=1\right\}$, which represents the outage event at the relay, can be found following the same steps used to obtain $\mathbb{P}_{\Theta|\varepsilon_0}\left\{\mathcal{A}_{DT}(R)|\varepsilon_0=1\right\}$. ■

It is important to notice that the term which considers the outage at the relay:

$$p_r\left(1-\exp\left\{-\lambda_s\delta\|r\|^2\left(1+\frac{2p_r}{\alpha}\right)\right\}\right), \quad (83)$$

is not a concave function of p_r. In this way, we cannot be sure that the upper bound to $\mathbb{P}_{\text{out,mix}}(R,p_r)$ in (81) is concave in p_r. However, our numerical results indicate that at least when $\lambda_s\delta\max\left\{\|r-d\|^2, D^2\right\} \leq 0.38$, that seems to be true. In fact, it is easy to see that for all p_r:

$$0 \leq p_r\left(1-\exp\left\{-\lambda_s\delta\|r\|^2\left(1+\frac{2p_r}{\alpha}\right)\right\}\right) \leq 1-\exp\left\{-\lambda_s\delta\|r\|^2\left(1+\frac{2p_r}{\alpha}\right)\right\}, \quad (84)$$

that is, the upper bound in (81) to $\mathbb{P}_{\text{out,mix}}(R,p_r)$ can be tightly bounded between 2 concave functions of p_r. Notice that the tightness is given by the quality of the link source-relay. As long as $\lambda_s\delta\|r\|^2$ is small enough, the bounds in (84) will be very tight and the upper bound,

and it is very likely that $\mathbb{P}_{\text{out,mix}}(R, p_r)$ is a concave function. Again, notice that $\lambda_s \delta \|r\|^2$ small is coherent with the use of the path loss approximation in IV-A and the union bound. In this way, the results of corollary 4.1 should also apply to this case under the conditions mentioned above.

In order to obtain more insightful results let us introduce:

$$\Delta(p_r) = \delta \left(1 + \frac{2p_r}{\alpha} \right). \tag{85}$$

In general we shall be interested in $\Delta(1) = \delta \left(1 + \frac{2}{\alpha}\right)$, so unless we specify the value of p_r we shall simply write $\Delta \equiv \Delta(1)$. Now lets assume that $\lambda_s \Delta = \lambda_s \delta \left(1 + \frac{2}{\alpha}\right)$ is small. As mentioned before, this will be in general the typical regime of operation. Usually $\|r\| \leq D$ and $\|r - d\|$ will be comparable to D. These observations will allow us to have the following result:

Lemma 4.1 (SND regime for fixed but non-collocated relays): In the SND regime we have:

$$\kappa(R) \leq m(p_r), \tag{86}$$

where:

$$\begin{aligned} m(p_r) = \frac{2\delta}{\alpha\pi} \left(\frac{\|r\|^2}{D^2} - D^{\alpha-2} \frac{D^2 - \|r-d\|^2}{D^\alpha - \|r-d\|^\alpha} \right) p_r^2 \\ + \frac{\delta}{\pi} \left(\frac{\|r\|^2}{D^2} + \frac{2}{\alpha} - D^{\alpha-2} \frac{D^2 - \|r-d\|^2}{D^\alpha - \|r-d\|^\alpha} \right) p_r + \frac{\delta}{\pi}. \end{aligned} \tag{87}$$

Proof: Follows from the fact that $e^{-u} = 1 - u + O(u^2)$. ■

In the small node density regime we can use the function $m(p_r)$ to study regions in which $p_r = 1$ is optimal. For that reason we have the following lemma proved in appendix C-B:

Lemma 4.2 (Region of optimality of $p_r = 1$ for fixed relays): The function $m(p_r)$ presents the following properties:

- $\forall p_r \in [0,1]$ it is concave if the relay relative position r is contained in:

$$A(\alpha, d) = \bigcup_{x \geq 0} \left\{ r \in \mathbb{R}^2 : \|r\|^2 \leq D^2 \frac{1-x}{1-x^{\frac{\alpha}{2}}}, \ \ \|r-d\|^2 = xD^2 \right\}. \tag{88}$$

- The value $p_r = 1$ is optimal if the relay relative position r is contained in:

$$B(\alpha, d) = \bigcup_{x \geq 0} \left\{ r \in \mathbb{R}^2 : \|r\|^2 \leq D^2 \left[\frac{1-x}{1-x^{\frac{\alpha}{2}}} - \frac{2}{\alpha+2} \right], \ \ \|r-d\|^2 = xD^2 \right\}. \tag{89}$$

It is interesting to observe that $B(\alpha, d) \subset A(\alpha, d)$, which implies that in a practical setting, the set of $r \in \mathbb{R}^2$ where $p_r = 1$ is optimal is contained in the region where $m(p_r)$ is concave.

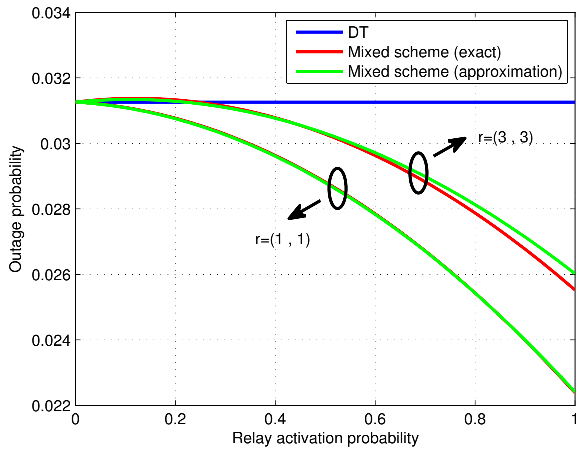


Fig. 10. Outage probability $\mathbb{P}_{\text{out,mix}}(R, p_r)$ as a function of p_r when $r = (1,\ 1)$ and $r = (3,\ 3)$. The other parameters are $d = (10,\ 0)$, $\lambda_s = 10^{-4}$, $R = 0.5$.

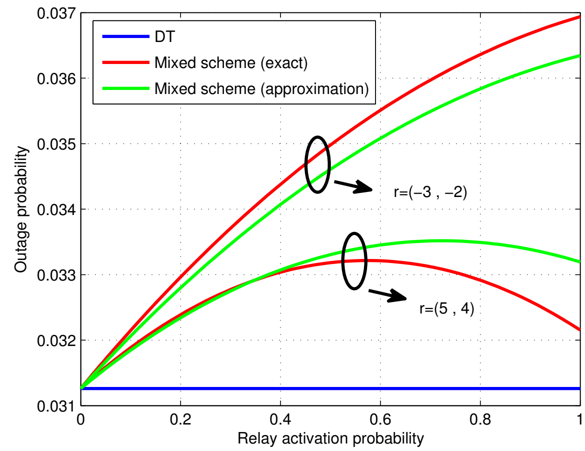


Fig. 11. Outage probability $\mathbb{P}_{\text{out,mix}}(R, p_r)$ as a function of p_r when $r = (5,\ 4)$ and $r = (-3,\ -2)$. The other parameters are $d = (10,\ 0)$, $\lambda_s = 10^{-4}$, $R = 0.5$.

We also see that $\lim_{\alpha\to\infty} B(\alpha, d) = A(\alpha, d)$ implying that when the path loss exponent is large enough the two sets coincide.

Remark 4.3: It can be shown that the unions in the (88) and (89) can be restricted to corresponding compact sets in $\mathbb{R}^2$ and not all $x \geq 0$. It can also be shown that both regions $B(\alpha, d)$, $A(\alpha, d)$ are compact subset in $\mathbb{R}^2$.

In Fig. 10 and 11, we see the outage probability $\mathbb{P}_{\text{out,mix}}(R, p_r)$ versus p_r for different settings and using the numerically calculated exact outage probability (59) and the approximate one in (81). In all cases it is important to mention how tight is the approximate expression. We see that in some cases, the approximate expression in (81) is not a strict upper bound to the true $\mathbb{P}_{\text{out,mix}}(R, p_r)$ in (59). This is because of the path loss approximation discussed in subsection IV-A. In any case the curves are tight enough to be useful approximations to the true $\mathbb{P}_{\text{out,mix}}(R, p_r)$. From the figures we can also see that when the relay is sufficiently far from the source, the performance of the mixed scheme degrades with respect to DT. The same happens when the relay is located near the source but “behind” it. In the first case, the reason from the degradation is from the fact that the relays are far from the sources and cannot decode the message reliably. In this way, interference is introduced in the network due to the relays presence, but without enjoying any benefit from cooperation. In the second case, the relays are

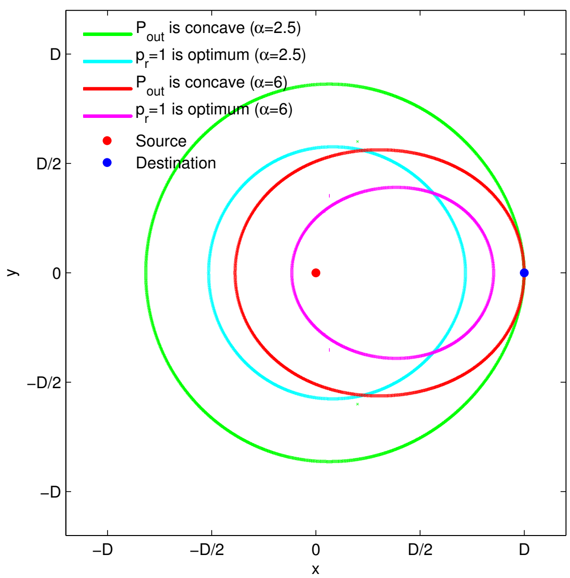


Fig. 12. Regions of concavity and optimality of $p_r = 1$ using $m(p_r)$ in (87) for $\alpha = 2.5$ and $\alpha = 6$.

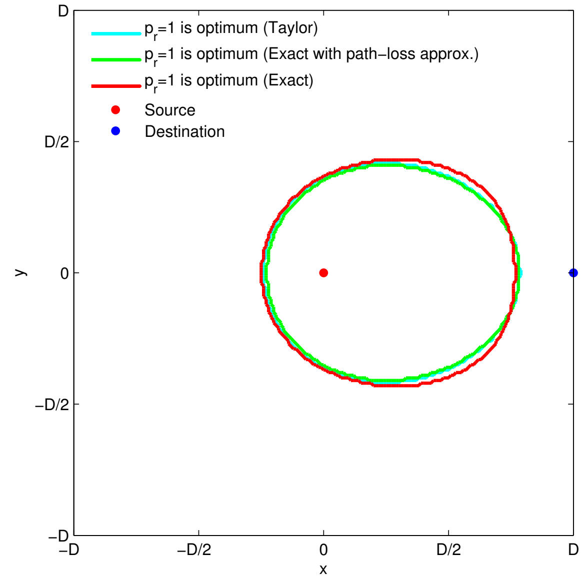


Fig. 13. Regions of optimality of $p_r = 1$ calculated using the exact outage probability (59), the approximate expression (81) and function $m(p_r)$. $d = (10,\ 0)$, $\lambda_s = 10^{-4}$, $\alpha = 4$, $R = 0.5$.

able to decode reliably, but the cooperative gain in the second term of (14) is not sufficient to deal with the aggregate interference generated by the relays. This is because, the quality of the link relay-destination is not good enough to provide a boost in performance due to cooperation.

In Fig. 12 we see the regions where $m(p_r)$ is concave and where $p_r = 1$ is optimal for that function, for two different values of α. We see that that as α is increased, the source is displaced from the center of the regions and is positioned in a corner. In the same manner, the regions are expanded towards the destination. This figure clearly shows that the best relay positions (where $p_r = 1$ is optimal) are biased in the direction toward the destination. Positioning the relays "behind" the source is not the best, specially when the path loss exponent α is large enough. This result agrees with the intuition about the problem. In Fig. 13 we see the regions of optimality of $p_r = 1$ obtained using the numerical exact outage probability (59), the approximate expression (81) and function $m(p_r)$. We see the excellent agreement of the obtained simplified expressions with the true behavior given by the exact expressions.

E. Relays Randomly Located Around Their Sources

In this section we assume that the potential relays are distributed according to the NN distribution around each source. This means that in each cluster the relay will be located at a different position. To derive the OP we have to average with respect to ε_0 as we did in previous sections, and also with respect to the position of the NN of the origin, r. As we have done so far, we shall consider that $\rho = 0$, and we shall use the union bound to obtain expressions amenable for analysis. Using these hypothesis the OP can be written as:

$$\mathbb{P}_{\text{out,mix}}(R) \leq \mathbb{P}_{\varepsilon_0}\left\{\varepsilon_0 = 1\right\}\left(\mathbb{E}_r\left[\mathbb{P}_{\Theta|\varepsilon_0,r}\left\{\mathcal{A}(R)|\varepsilon_0 = 1\right\}\right] + \mathbb{E}_r\left[\mathbb{P}_{\Theta|\varepsilon_0,r}\left\{\mathcal{B}(R)|\varepsilon_0 = 1\right\}\right]\right)$$
$$+ \mathbb{P}_{\Theta}\left\{\mathcal{A}_{DT}(R)|\varepsilon_0 = 0\right\}\mathbb{P}_{\varepsilon_0}\left\{\varepsilon_0 = 0\right\}, \quad (90)$$

where we have used that r is independent of ε_0, and that $\mathcal{A}_{DT}$ is independent of r. The following Theorem deals with the expression of the OP and the SND regime:

Theorem 4.6 (OP for random relays and the SND regime): The OP when the relays are selected as the nearest neighbors of each source can be upper bounded as:

$$\mathbb{P}_{\text{out,mix}}(R) \leq (1-p_r)\left[1 - e^{-\lambda_s\Delta(p_r)D^2}\right] + p_r\left\{1 + \frac{\lambda_s\Delta(p_r)}{\pi\lambda_{in} + \lambda_s\Delta(p_r)} - \right.$$
$$\left. e^{-\lambda_s\Delta(p_r)D^2}\left[1 + \lambda_s\Delta(p_r)D^2\left(1 + \frac{2-\alpha}{\alpha D}\mathbb{E}_r[||r-d||]\right) + O\left((\lambda_s\Delta(p_r)D^2)^2\right)\right]\right\} \quad (91)$$

as $\lambda_s\Delta(p_r)D^2 \equiv \lambda_s\delta\left(1 + \frac{2p_r}{\alpha}\right)D^2 \to 0$. In the SND regime we have:

$$\kappa(R) \leq \frac{\delta}{\pi}\left(1 + \frac{2p_r}{\alpha}\right)\left[1 - p_r\left(1 - \frac{2\sigma_{in}^2}{D^2} - \left(1 - \frac{2}{\alpha}\right)\frac{\mathbb{E}_r\left[||r-d||\right]}{D}\right)\right]. \quad (92)$$

The expectation of $||r-d||$ is given by (55) or can be upper bounded by (56).

Proof: For the first part see appendix C-C and the second part as usual follows from the fact that $e^u = 1 + u + O(u^2)$. ■

An important issue in this setup is to study the optimal relay activation probability, since now each relay is randomly located. In the previous section the optimal choice of p_r depended on the set of parameters (α, d, r, λ_s, R). In the random relay setup, the position of the relay is different in each cluster. Hence, the optimal relay activation probability will depend on the set of parameters (α, d, σ_{in}, λ_s, R), that is, it will now depend on the average distance to the

NN with respect to the transmission distance D. In what follows we shall neglect the term $O\left((\lambda_s\Delta(p_r)D^2)^2\right)$ in (91) which is a natural approach when the OP is small.

Theorem 4.7 (Concavity of the OP for random relays): When the OP is small, neglecting the term $O\left((\lambda_s\Delta(p_r)D^2)^2\right)$ in (91), for each value of $\lambda_s\delta D^2$ there exists an interval $0 \leq \sigma_{in} \leq \sigma_{in,c}(D, \lambda_s\delta D^2, \alpha)$ such that the OP is a concave function of $p_r \in [0,1]$.

In addition if $\lambda_s\delta D^2 \leq 0.38$ then a lower bound of $\sigma_{in,c}(D, \lambda_s\delta D^2, \alpha)$ is obtained by solving the equation:

$$e^{\lambda_s\Delta D^2}\left(2\alpha s^2 + (\alpha-2)\gamma(s)s\right) - (\lambda_s\Delta D^2)^2 + (4+\lambda_s\delta D^2)\lambda_s\Delta D^2 - (2+3\lambda_s\delta D^2) = 0, \quad (93)$$

where $s = \sigma_{in}/D$. Finally a closed form lower bound of $\sigma_{in,c}$ is given by:

$$\sigma_{in,c} \geq \sqrt{\frac{(\lambda_s\Delta D^2)^2 - (4+\lambda_s\delta D^2)\lambda_s\Delta D^2 + (2+3\lambda_s\delta D^2)}{2\alpha}e^{-\frac{1}{2}\lambda_s\Delta D^2} - \frac{(\alpha-2)}{8\alpha}}. \quad (94)$$

Proof: See appendix C-D. ■

Finally the following Theorem deals with sufficient conditions for $p_r = 1$ to be the optimal relay activation probability.

Theorem 4.8 (Optimality region of $p_r = 1$ for random relays): When the OP is small, neglecting the term $O\left((\lambda_s\Delta(p_r)D^2)^2\right)$ in (91), for each value of $\lambda_s\delta D^2$ there exists an interval $0 \leq \sigma_{in} \leq \sigma_{in,1}(D, \lambda_s\delta D^2, \alpha)$ such that the optimal relay activation probability is $p_r = 1$.

In addition if $\lambda_s\delta D^2 \leq 0.38$ then a lower bound of $\sigma_{in,1}(D, \lambda_s\delta D^2, \alpha)$ is obtained by solving the equation:

$$2\alpha\left(\alpha+2\right)e^{\lambda_s\Delta D^2}s^2 + \left(\alpha^2-4\right)\gamma(s)s + 4\left(\frac{2}{3}\lambda_s\delta D^2 - 1\right) = 0, \quad (95)$$

where $s = \sigma_{in}/D$. Finally a closed form lower bound of $\sigma_{in,\max}$ is given by:

$$\sigma_{in,1} \geq \sqrt{\frac{2\left(1-\frac{2}{3}\lambda_s\delta D^2\right)}{\alpha(\alpha+2)}e^{-\frac{1}{2}\lambda_s\Delta D^2} - \frac{(\alpha-2)}{10\alpha}e^{-\lambda_s\Delta D^2}}. \quad (96)$$

Proof: See appendix C-E. ■

To finish this section we present a few figures to study the expressions we just derived. In Fig. 14 the OP with respect to p_r is plotted for two different values of σ_{in}, one in which $p_r = 1$ is optimal and another one for which $p_r = 0$ is the optimal point. The approximations of the OP come from using (55) and (56) in (91) and they are compared with Montecarlo simulations obtained by averaging 8×10^6 realizations of the PPP using the true interferences (5) and (6),

taking $d = (10,\ 0)$, $\lambda_s = 10^{-4}$, $R = 0.5$. We see that the approximations derived with the simplified interferences are in excellent agreement with the actual OP derived with the more complex interferences.

In Fig. 15 we study the maximum values of σ_{in}/D for the concavity of the OP and for $p_r = 1$ to be optimal in the SND regime. We consider $\alpha = 3$, $d = (10,\ 0)$ and $R = 0.5$. Numerical regions are derived by numerical integration of (81), while the numerical roots and the closed form plots of the concavity region come from (93) and (94). When considering the optimality of $p_r = 1$, the numerical roots are obtained by solving (95) and the closed form expression through (96). We observe that the conditions derived in Theorems 4.7 and 4.8 predict very well the actual concavity regions and regions of optimality of $p_r = 1$. In addition, the simple closed form expressions are very close to the numerical roots.

Finally in Fig. 16 we study the maximum rate achievable for the mixed scheme with $p_r = 1$ for a fixed OP constraint, relative to the DT rate obtained for the same OP. This is equivalent to comparing the scenario in which all clusters use DF ($p_r = 1$) with the scenario of using DT ($p_r = 0$). When the ratio of the maximum rates falls below one, then $p_r = 0$ is better than $p_r = 1$. We consider $\lambda_s = 10^{-4}$, $d = (10, 0)$ and $\alpha = 4$. The rates come from using (55) in (91). We see that DF provides increasing gains as the relays become more concentrated around their sources. However, these gains are smaller than the ones observed in section III-B where only the source at the origin had a relay. This shows that the aggregate interference introduced in the network by the cooperation can reduce its benefits significantly.

Finally in Fig. 17 we see the relative reduction of $\kappa(R)$ with respect to $\kappa_{\mathrm{DT}}(R)$ as function of σ_{in}/D for several cases of interest[8]. In particular we consider the case of $\kappa_{\mathrm{mix}}(R)$ which corresponds to the mixed randomized protocol analyzed above, the case of $\kappa_{\mathrm{DF}}(R)$ which corresponds to the case in which only the source at the origin has a relay associated (section III-B) and $\kappa_{\mathrm{CS}}(R)$ (calculated for the case in which all the nodes in the network can have an associated relay) which corresponds to the asymptotic value of the lower on the asymptotic error probability in (11). Several comments are in order. The value of $\kappa_{\mathrm{DF}}(R)$ presents important reduction with respect to the case of $\kappa_{\mathrm{DT}}(R)$. In fact, for $\sigma_{in} = 0$ presents a reduction of 100

[8]We are making an abuse of notation here, denoting as $\kappa(R)$, the quantities plotted in Fig. 17. In fact they are only the upper bounds the corresponding upper and lower bounds derived in the previous sections using the approximations obtained for the OP.

% with respect to $\kappa_{\mathrm{DT}}(R)$. In this situation, the relay associated with the source at the origin physically collocated with it. This is equivalent to a MISO system with two transmitting antennas. The plot and the definition of $\kappa(R)$ in equation (12) suggests that the gain observed is similar in nature to a diversity gain of 2 [25]. As σ_{in} is increased, we see that the gain diminishes, as the second antenna (the relay), cannot decode for free the source message, in order to achieve the second term in (14). The behaviour of $\kappa_{\mathrm{mix}}(R)$ is similar. However, as in this case all sources in the network can have an associated relay, the gains are smaller. This is because, a given source-relay-destination triplet has to deal with more interference. The difference between the curves corresponding to $\kappa_{\mathrm{DF}}(R)$ and $\kappa_{\mathrm{mix}}(R)$ can be interpreted as the penalty of allowing cooperation network-wide. We see that, when $\sigma_{in}/D \approx 0.27$, there is no reduction of $\kappa_{\mathrm{mix}}(R)$ with respect to the baseline of $\kappa_{\mathrm{DT}}(R)$. This is because in that case the optimal value of p_r is zero, and cooperation in not longer beneficial for the network. The value of $\kappa_{\mathrm{CS}}(R)$, as obtained from a lower bound to the asymptotic error probability, presents a better behaviour with respect to the baseline. It is important to notice that when σ_{in} is small, the value of $\kappa_{\mathrm{mix}}(R)$ is near optimal. However, as σ_{in} increases there is a rapid degradation of $\kappa_{\mathrm{mix}}(R)$ with respect to the optimal value.

V. Exploiting CSI at the Relays

In the above analysis we assumed that each relay decides to activate itself with probability p_r independent of all network parameters, which gives rise to a randomized mixed scheme. However, a better strategy would be to let the relays decide when it would be convenient to help their corresponding sources based on some local knowledge they could have. Given that each relay has full CSI of the corresponding link with its source, the relays could determine whether they would be able to decode the message from the source or not. In the latter case, the relay can communicate his impairment to the destination (at a negligible communication cost) and avoid the retransmission of an erroneous message and the corresponding aggregate interference to the network. In this way the destination will try to decode the message using only the transmission from the source node, which reduces the scheme to DT. Clearly, this scheme can not be worse than DT. The corresponding OP of this scheme, which uses the full CSI information at the relay,

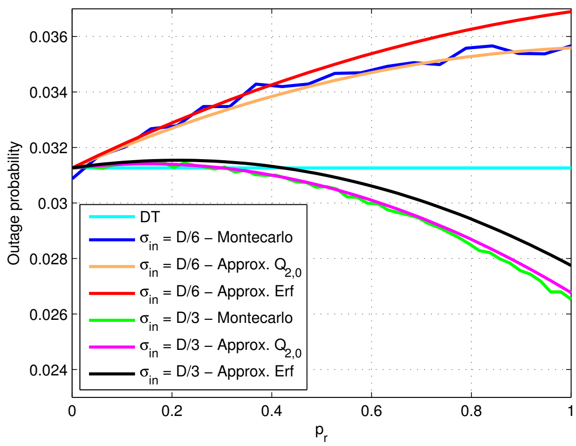


Fig. 14. Outage probability $\mathbb{P}_{\text{out,mix}}(R, p_r)$ as a function of p_r for values of σ_{in} showing optimality of $p_r = 0$ or $p_r = 1$. $d = (10,\ 0)$, $\lambda_s = 10^{-4}$, $R = 0.5$. Montecarlo simulations are obtained by averaging 8×10^6 realizations of the PPP using (5) and (6). Approximations come from using (55) and (56) in (91).

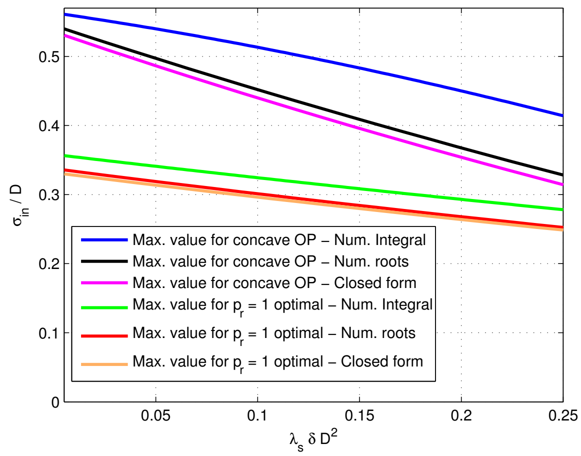


Fig. 15. Maximum values of σ_{in}/D for concavity of the OP and for $p_r = 1$ to be optimal. $\alpha = 3$, $d = (10,\ 0)$, $R = 0.5$. Numerical integrals come from numerical integration of (81). The numerical roots come and the closed form plots of the concavity region come from (93) and (94). The numerical roots come and the closed form plots of the optimality of $p_r = 1$ come from (95) and (96).

can be written as[9]:

$$\mathbb{P}_{\text{out, FCSI}}\left(R\right) = \inf_{\rho\in\mathbb{C},|\rho|\leq 1} \mathbb{P}_{\Theta}\left\{\mathcal{B}(R,\rho)\cap\mathcal{A}^{c}(R,\rho)\right\} + \mathbb{P}_{\Theta}\left\{\mathcal{A}_{DT}\left(R\right)\cap\mathcal{A}\left(R,\rho\right)\right\} \tag{97}$$

It is important to write the values of the interference power at the relay and destination associated with the source at the origin:

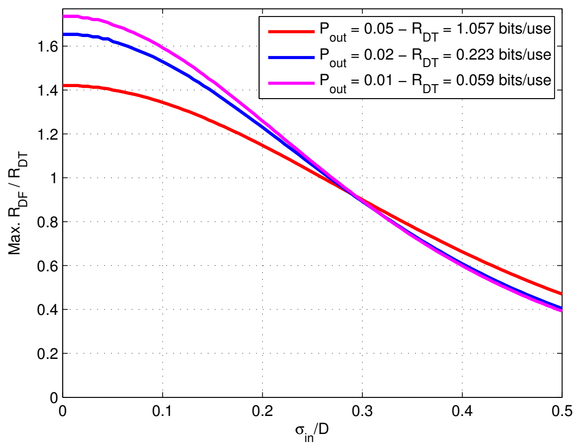
$$I_r = \sum_{i:x_i\in\Phi_s}\left[\frac{|h_{x_i r}|^2}{\|x_i - r\|^{\alpha}} + \mathbb{1}\left\{\mathcal{A}^{c}_{k_i}\left(R,\rho\right)\right\}\left(\frac{|h_{k_i r}|^2}{\|x_i + k_i - r\|^{\alpha}} + \frac{2\Re\left\{h_{x_i r}h^{*}_{k_i r}\rho\right\}}{\|x_i - r\|^{\frac{\alpha}{2}}\|x_i + k_i - r\|^{\frac{\alpha}{2}}}\right)\right], \tag{98}$$

$$I_d = \sum_{i:x_i\in\Phi_s}\left[\frac{|h_{x_i d}|^2}{\|x_i - d\|^{\alpha}} + \mathbb{1}\left\{\mathcal{A}^{c}_{k_i}\left(R,\rho\right)\right\}\left(\frac{|h_{k_i d}|^2}{\|x_i + k_i - d\|^{\alpha}} + \frac{2\Re\left\{h_{x_i d}h^{*}_{k_i d}\rho\right\}}{\|x_i - d\|^{\frac{\alpha}{2}}\|x_i + k_i - d\|^{\frac{\alpha}{2}}}\right)\right], \tag{99}$$

where $\mathcal{A}_{k_i}\left(R,\rho\right)$ denotes the event of decoding failure at the relay associated with node x_i:

$$\mathcal{A}_{k_i}\left(R,\rho\right) = \left\{\frac{|h_{s_i k_i}|^2}{I_{k_i}\|k_i\|^{\alpha}} < T\right\}. \tag{100}$$

[9]It should be clear that in this case Θ does not include δ_0 and $\tilde{\Phi}_s$ does not include the marks δ_i, because the activation of the relay is done with another mechanism.



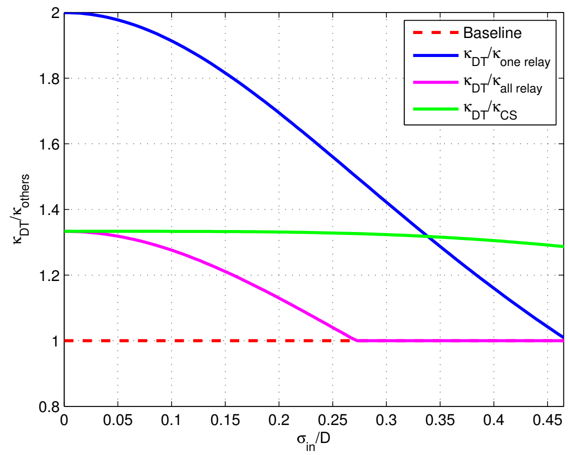


Fig. 16. Maximum rate for the mixed scheme with $p_r = 1$, relative to the same rate of DT for a fixed OP. The mixed scheme rates are obtained by using (55) in (91). $\lambda_s = 10^{-4}$. $d = (10, 0)$, $\alpha = 4$.

Fig. 17. Relative reduction of $\kappa_{\mathrm{mix}}(R)$, $\kappa_{\mathrm{CS}}(R)$ and $\kappa_{\mathrm{DF}}(R)$ (only source at the origin has a relay) with respect to $\kappa_{\mathrm{DT}}(R)$ as function of σ_{in}/D . $R = 0.5$, $\lambda_s = 10^{-4}$. $d = (10, 0)$, $\alpha = 4$.

From the previous equation it is observed that the interference power at a given position in space depends explicitly on the interference at the points of space where a relay can be positioned. That is, in order to obtain the interference power at a given position, we need to know the interference power at all relay positions. This could be very difficult to solve, and from a practical point view a given relay decision would be linked to all the others relays ones. If a given relay decides not to transmit, this would affect the decision of other relays in the network. This means that the use of full CSI at each relay leads to a transmission scheme that in essence requires some kind of network-wide consensus, which could be impossible to achieve in practice. For this reason we will propose a scheme which use partial CSI at the relays. In precise terms, the relay associated with the source at the origin, will consider the following set:

$$\mathcal{D}\left(\xi\right) = \left\{ \frac{|h_{sr}|^2}{\|r\|^{\alpha}} \geq \xi \right\}, \tag{101}$$

where ξ is an appropriately chosen threshold. When $\mathcal{D}\left(\xi\right)$ happens the relay will try to decode the message, because the event $\mathcal{D}\left(\xi\right)$ indicates that the fading and path loss gains of its link with the source node are strong enough. The outage probability of this scheme can be written

as:

$$\mathbb{P}_{\text{out, PCSI}}\left(R\right)=\inf_{\rho\in\mathbb{C},|\rho|\leq 1}\mathbb{P}_{\Theta}\left\{\left(\mathcal{B}(R,\rho)\cup\mathcal{A}(R,\rho)\right)\cap\mathcal{D}\left(\xi\right)\right\}+\mathbb{P}_{\Theta}\left\{\mathcal{A}_{DT}\left(R\right)\cap\mathcal{D}^{c}\left(\xi\right)\right\} \quad (102)$$

The expressions for the interference powers at the relay and destination associated with the source at the origin are given by (98) and (99) with $\mathbb{1}\left\{\mathcal{A}_{k_i}\left(R,\rho\right)\right\}$ replaced by $\mathbb{1}\left\{\mathcal{D}_{k_i}\left(\xi\right)\right\}$, where $\mathcal{D}_{k_i}\left(\xi\right)$ is the analogous to (101) with the relay associated with source x_i. Notice that as the events $\mathcal{D}_{k_i}\left(\xi\right)$ are not taking into account the possible impairment of the interference, the use of $\mathcal{D}_{k_i}\left(\xi\right)$ does not guarantee that the relays will decode their sources messages correctly, and this scheme does not necessarily degrade to DT as the one that uses full CSI. However, although the decisions of the relays based on $\mathcal{D}_{k_i}\left(\xi\right)$ are not fully randomized due to the fact that some information is used, they are decoupled. Besides that, they are statistically independent because of the independence between marks k_i and $h_{s_ik_i}$. This means that the interferences observed at the relay and destination associated for example with the source at the origin are statistically equivalent to the randomized mixed scheme of the previous sections with p_r chosen equal to $\mathbb{P}_{\Theta}\left\{\mathcal{D}\left(\xi\right)\right\}=e^{-\xi\|r\|^{\alpha}}$. However, the outage performance does not need to be the same as the randomized mixed scheme one. This is because, the sets $\mathcal{D}_{k_i}\left(\xi\right)$ have some information about the decoding capabilities of the relays. In fact, it can be shown that this scheme is better than the randomized mixed scheme when the interference levels are the same to the relays and destinations in both protocols, or what is the same, when $p_r=\mathbb{P}_{\Theta}\left\{\mathcal{D}_{k_i}\left(\xi\right)\right\}$. In order to prove this result notice that because of the independence among the fading coefficients we have:

$$\mathbb{P}_{\Theta}\left\{\mathcal{A}_{DT}\left(R\right)\cap\mathcal{D}^{c}\left(\xi\right)\right\}=\mathbb{P}_{\Theta}\left\{\mathcal{A}_{DT}\left(R\right)\right\}\mathbb{P}_{\Theta}\left\{\mathcal{D}^{c}\left(\xi\right)\right\}. \quad (103)$$

We can also write:

$$\begin{aligned}\mathbb{P}_{\Theta}\left\{\left(\mathcal{B}(R,\rho)\cup\mathcal{A}(R,\rho)\right)\cap\mathcal{D}\left(\xi\right)\right\}=\mathbb{P}_{\Theta}&\left\{\left(\mathcal{B}(R,\rho)\cap\mathcal{D}\left(\xi\right)\right)\right\}\\&+\mathbb{P}_{\Theta}\left\{\mathcal{A}(R,\rho)\cap\mathcal{D}\left(\xi\right)\cap\mathcal{B}^{c}(R,\rho)\right\}. \quad (104)\end{aligned}$$

It is straightforward to show that:

$$\mathbb{P}_{\Theta}\left\{\mathcal{B}\left(R\right)\cap\mathcal{D}\left(\xi\right)\right\}=\mathbb{P}_{\Theta}\left\{\mathcal{B}\left(R\right)\right\}\mathbb{P}_{\Theta}\left\{\mathcal{D}\left(\xi\right)\right\}. \quad (105)$$

Let us analyze the term $\mathbb{P}_{\Theta}\left\{\mathcal{A}(R,\rho)\cap\mathcal{D}\left(\xi\right)\cap\mathcal{B}^{c}(R,\rho)\right\}$. It can be easily shown that this term contains the effect of the partial CSI at the relay, and how the use of that information could improve the outage probability performance. We have the following lemma:

Lemma 5.1: For every $\xi \geq 0$ the following holds:

$$\mathbb{P}_\Theta\{\mathcal{A}(R,\rho) \cap \mathcal{D}(\xi) \cap \mathcal{B}^c(R,\rho)\} \leq \mathbb{P}_\Theta\{\mathcal{A}(R,\rho) \cap \mathcal{B}^c(R,\rho)\}\, \mathbb{P}_\Theta\{\mathcal{D}(\xi)\} \tag{106}$$

Proof: Notice that we can write:

$$\mathbb{P}_\Theta\{\mathcal{A}(R,\rho) \cap \mathcal{D}(\xi) \cap \mathcal{B}^c(R,\rho)\} = \mathbb{E}_{\tilde{\Phi},r}\left[\mathbb{P}_{\Theta|\tilde{\Phi}_s}\left\{\mathcal{A}(R,\rho) \cap \mathcal{D}(\xi) \cap \mathcal{B}^c(R,\rho) \middle| \tilde{\Phi}_s, r\right\}\right]. \tag{107}$$

Conditioned on $\tilde{\Phi}_s$ it is clear that $\mathcal{A}(R,\rho) \cap \mathcal{D}(\xi)$ is independent of $\mathcal{B}^c(R,\rho)$. Using this fact jointly with the results from subsection III-A we can write:

$$\mathbb{P}_\Theta\{\mathcal{A}(R,\rho) \cap \mathcal{D}(\xi) \cap \mathcal{B}^c(R,\rho)\} = \mathbb{E}_{\tilde{\Phi}_s,r}\left[\mathbb{P}_{\Theta|\tilde{\Phi}_s}\left\{\mathcal{A}(R,\rho) \cap \mathcal{D}(\xi) \middle| \tilde{\Phi}_s, r\right\} \bar{F}_V\left(TI_d^\xi\right)\right], \tag{108}$$

where we have denoted with I_d^ξ the interference at the destination of the source in the origin when the threshold is ξ. It can also be shown that:

$$\mathbb{P}_{\Theta|\tilde{\Phi}_s,r}\left\{\mathcal{A}(R,\rho) \cap \mathcal{D}(\xi) \middle| \tilde{\Phi}_s\right\} = \begin{cases} e^{-\xi\|r\|^\alpha} - e^{-\frac{TI_r^\xi\|r\|^\alpha}{1-|\rho|^2}} & \xi \leq \frac{TI_r^\xi}{1-|\rho|^2} \\ 0 & \text{otherwise} \end{cases} \tag{109}$$

In this way we can write:

$$\begin{aligned}\mathbb{P}_\Theta\{\mathcal{A}(R,\rho) \cap \mathcal{D}(\xi) \cap \mathcal{B}^c(R,\rho)\} &= e^{-\xi\|r\|^\alpha} \\ &\times \mathbb{E}_{\tilde{\Phi}_s,r}\left[\mathbb{1}\left\{\xi \leq \frac{TI_r^\xi}{1-|\rho|^2}\right\}\left(1 - e^{\xi\|r\|^\alpha - \frac{TI_r^\xi\|r\|^\alpha}{1-\|rho|^2}}\right)\bar{F}_V\left(TI_d^\xi\right)\right].\end{aligned} \tag{110}$$

Using the fact that:

$$\mathbb{P}_\Theta\{\mathcal{A}(R,\rho) \cap \mathcal{B}^c(R,\rho)\} = \mathbb{E}_{\tilde{\Phi}_s,r}\left[\left(1 - e^{-\frac{TI_r^\xi\|r\|^\alpha}{1-|\rho|^2}}\right)\bar{F}_V\left(TI_d^\xi\right)\right], \tag{111}$$

and that for all $\xi, I_r^\xi \geq 0$:

$$\mathbb{1}\left\{\xi \leq \frac{TI_r^\xi}{1-|\rho|^2}\right\}\left(1 - e^{\xi\|r\|^\alpha - \frac{TI_r^\xi\|r\|^\alpha}{1-\|rho|^2}}\right) \leq 1 - e^{-\frac{TI_r^\xi\|r\|^\alpha}{1-|\rho|^2}}, \tag{112}$$

the result in (106) follows straightforwardly. ■

We have then the following corollary, which follows directly from the above lemma:

Corollary 5.1: Assume that $p_r = \mathbb{P}_\Theta\{\mathcal{D}(\xi)\} = e^{-\xi\|r\|^\alpha}$. Then we have:

$$\mathbb{P}_{\text{out, PCSI}}(R) \leq \mathbb{P}_{\text{out,mix}}(R). \tag{113}$$

This result shows that the scheme using partial CSI has a better performance than the mixed randomized scheme analyzed in the previous sections, when the ratio of active relays is the same in both (measured by p_r and $\mathbb{P}_\Theta \{\mathcal{D}(\xi)\}$ respectively) or what is the same, when both schemes are subjected to the same interference levels. We should mention, however, that our numerical results (not shown) suggest that the performance gains by using partial CSI are not significant with respect to the mixed randomized scheme. Moreover, it seems that the performance of the scheme using partial CSI is extremely sensitive to the parameter ξ, whose optimal value cannot be computed in closed form and depends on all the network parameters.

VI. Summary and Final Remarks

In this work we presented an analysis of the overall balance of cooperation and interference generation in large interference limited wireless networks. The emphasis was put on networks where the relays are located in the proximity of their sources. This naturally leads to the use of the DF scheme in order to obtain the best possible benefits from cooperation. In fact, the principal scheme analyzed in this paper is a mixed randomized one in which DT or DF can be used. The choice between DT and DF is done by the corresponding relay associated with each source via a randomized decision with probability p_r and without taking into account any piece of information the relays could have. This simple procedure, which is mathematically tractable, can be thought as a MAC layer at the relays (in a similar fashion as the popular ALOHA protocol), with the objective of limiting the interference generation in the network. With this simple model, a balance between cooperation and interference generation can be established in the network. Surprisingly, for several cases of interest, and for typical operating conditions, the optimal values of p_r are 0 or 1, revealing a binary behavior: the best is that all nodes in the network enjoy the benefit of cooperation or none at all. There is not an optimal ratio of active aiding relays in the network. This means that cooperation could be beneficial or detrimental to all the users in the network. Even when cooperation is beneficial to all, the performance improvements are not as large in the typical fading relay channel with Gaussian noise. The reason of this comes from the fact that, in addition to fading, there is an additional random component given by the network nodes positions. Moreover, as discussed in section V the use of partial CSI at the relays as guidance for their activation decision does not seem to offer additional improvements. And the use of full CSI, although surely more beneficial, could be very difficult to implement because

some kind of "network consensus" is required.

Notice that, despite the above results, it should not be implied that cooperation will not provide improvements in interference limited scenarios. The fact that relays and destinations treat all the interference from the other users as noise severely limits the benefits of cooperation for a given source-destination pair. A potential improvement could be obtained using more sophisticated cooperative transmission schemes which could take into account the impairments generated by the nearby interferers [5] (which introduce by far the most harmful interference). In such situation, besides the intrinsic benefits of cooperation, the smart use of the aggregate interference introduced in part by the cooperating nodes, could ameliorate its harmful effect on the overall network. Also, the study of other cooperative schemes as AF and CF deserves full consideration. All these issues, as well as the effect of using several potential relays instead of only one, constitute important and interesting future work directions.

APPENDIX A
INTERFERENCE RVS AND THEIR LAPLACE TRANSFORMS

This appendix is presented to simplify the use of the Laplace transform of interference RVs throughout this work. For a more detailed analysis see [13] and [14]. Let $\tilde{\Phi} = \{(x_i, m)\}$ be an independently marked HPPP with $\Phi = \{x_i\}$ an HPPP in $\mathbb{R}^2$ and m a vector of marks on a subset of $\mathbb{R}^l$, $l \geq 1$. Define the interference RVs:

$$I_d = \sum_{i:x_i \in \Phi_s} f_1(d, x_i, m_i) \qquad\qquad I_r = \sum_{i:x_i \in \Phi_s} f_2(r, x_i, m_i) \tag{114}$$

where f_1 and f_2 are real valued non negative functions. Then the joint Laplace transform of the interference RVs at point (ω_1, ω_2) is known to be [13]:

$$\mathcal{L}_{I_d,I_r}(\omega_1, \omega_2) = \exp\left\{-\lambda_s \int_{\mathbb{R}^2} \left(1 - \mathbb{E}_m\left[e^{-\omega_1 f_1(d,x,m) - \omega_2 f_2(r,x,m)}\right]\right) dx\right\}. \tag{115}$$

Taking $\omega_1 = 0$ or $\omega_2 = 0$ the single Laplace transforms are obtained.

Lemma A.1: When $f_1(d, x, m) = |h_1|^2 l(x, d)$ and $f_2(r, x, m) = |h_2|^2 l(x, r)$ with $|h_1|^2$ and $|h_2|^2$ independent exponential RVs with unit mean and $l(\cdot, \cdot)$ is the simplified path loss function, the joint Laplace transform (115) can be written as:

$$\mathcal{L}_{I_d,I_r}(\omega_1, \omega_2) = e^{-\lambda_s \left[C(\omega_1^{2/\alpha} + \omega_2^{2/\alpha}) + f(\omega_1, \omega_2)\right]}, \tag{116}$$

where $f(\omega_1, \omega_2)$ is given by (34), C is given by (35) and $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the usual Gamma function.

Proof: In this case $m = (|h_1|^2, |h_2|^2)$ so that:

$$\mathbb{E}_m \left[e^{-\omega_1 f_1(z_1,x) - \omega_2 f_2(z_2,x)} \right] = \frac{1}{(\omega_1 l(x, z_1) + 1)(\omega_2 l(x, z_2) + 1)}. \tag{117}$$

Replace (117) in (115) and factorize the integrand as:

$$1 - \frac{1}{(\omega_1 l(x, z_1) + 1)(\omega_2 l(x, z_2) + 1)} = \frac{1}{1 + \frac{1}{\omega_1} l(x, z_1)^{-1}} + \frac{1}{1 + \frac{1}{\omega_2} l(x, z_2)^{-1}} - \frac{1}{(1 + \frac{1}{\omega_1} l(x, z_1)^{-1})(1 + \frac{1}{\omega_2} l(x, z_2)^{-1})}. \tag{118}$$

For the simplified path loss function we use that:

$$\int_{\mathbb{R}^2} \frac{1}{1 + (\omega_1 l(x, r))^{-1}} dx = C \omega_1^{2/\alpha}, \tag{119}$$

a result which is known from the direct transmission case [3]. ■

Lemma A.2: Suppose the marks of the HPPP are $m = (|h_1|^2, |h_2|^2, \varepsilon, k)$, with $|h_1|^2$ and $|h_2|^2$ unit mean independent exponential RVs, ε a Bernoulli RV with success probability p_r, and k a RV on $\mathbb{R}^2$. Let $f_1(d, x, m) = (|h_1|^2 + \varepsilon |h_2|^2)\, l(x + \tau k, d)$ with $l(\cdot, \cdot)$ the simplified path loss function and $\tau \in [0, 1]$. Then the Laplace transform is:

$$\mathcal{L}_{I_d}(\omega_1) = \exp\left\{ -\lambda_s C \omega_1^{2/\alpha} \left(1 + \frac{2 p_r}{\alpha} \right) \right\}. \tag{120}$$

Proof: Take $\omega_2 = 0$ in (115) and compute the expectation with respect to the marks:

$$\mathbb{E}_m \left[e^{-\omega_1 f_1(d,x,m)} \right] = \int_{\mathbb{R}^2} \frac{p_r}{\left[1 + \omega_1 l(x + \tau k, d)\right]^2} dF_k + \int_{\mathbb{R}^2} \frac{1 - p_r}{1 + \omega_1 l(x + \tau k, d)} dF_k. \tag{121}$$

Replacing (121) in (115) and interchanging the integration order we find (115) yields:

$$\mathcal{L}_{I_d}(\omega_1) = \exp\left\{ -\lambda_s p_r \int_{\mathbb{R}^2} \left(\int_{\mathbb{R}^2} 1 - \frac{1}{\left[1 + \omega_1 l(x + \tau k, d)\right]^2} dx \right) dF_k - \lambda_s (1 - p_r) \int_{\mathbb{R}^2} \left(\int_{\mathbb{R}^2} \frac{1}{1 + (\omega_1 l(x + \tau k, d))^{-1}} dx \right) dF_k \right\}. \tag{122}$$

When the integrals with respect to x are computed the result does not depend on k so the distribution of k does not affect the final result. For the first integral we have:

$$\int_{\mathbb{R}^2} 1 - \frac{1}{\left[1+\omega_1 l(x+\tau k_i, d)\right]^2} dx = 2\pi\omega^{2/\alpha} \int_0^\infty \frac{1+2t^\alpha}{(1+t^\alpha)^2} t dt \tag{123}$$

$$= \omega_1^{2/\alpha}\left(C + 2\pi \int_0^\infty \frac{t^{\alpha+1}}{(1+t^\alpha)^2} dt\right), \tag{124}$$

$$= \omega_1^{2/\alpha}\left(1+\frac{2}{\alpha}\right) C. \tag{125}$$

For the last step we integrate by parts and C is defined on (35). The second integral is (119). ■

Appendix B

Proofs of section III

A. *Proof of lemma 3.1*

We need to prove that $\rho = 0$ minimizes the following function:

$$f(\rho) = \frac{\|r\|^2}{(1-|\rho|^2)^{2/\alpha}} + \frac{\mu_2^{1-\frac{2}{\alpha}} - \mu_1^{1-\frac{2}{\alpha}}}{\mu_2 - \mu_1}, \tag{126}$$

where μ_1 and μ_2 are given by (22) and (23). Since the first term is minimized when $\rho = 0$ we focus on the second one. Define:

$$a = l_{sd} + l_{rd}, \qquad b = \left[(l_{sd} - l_{rd})^2 + 4 l_{sd} l_{rd} |\rho|^2\right]^{1/2}. \tag{127}$$

so that we can write $\mu_1 = \frac{1}{2}(a-b)$ and $\mu_2 = \frac{1}{2}(a+b)$. Notice that $0 \leq b \leq a$, that ρ appears only in b and that b is increasing in $|\rho|$. Therefore it suffices to show that the auxiliary function:

$$g(b) = \frac{1}{2^{1-\frac{1}{\alpha}}} \frac{(a+b)^{1-\frac{2}{\alpha}} - (a-b)^{1-\frac{2}{\alpha}}}{b}. \tag{128}$$

is increasing in b to complete the proof. We differentiate to obtain:

$$\frac{dg}{db}(b) = \frac{1}{2^{1-\frac{1}{\alpha}}} \frac{(a-b)^{-\frac{2}{\alpha}}\left(a - \frac{2}{\alpha}b\right) - (a+b)^{-\frac{2}{\alpha}}\left(a + \frac{2}{\alpha}b\right)}{b^2}. \tag{129}$$

In order to conclude the proof we need to check that $dg/db(b) \geq 0$. That is:

$$t(b) \geq u(b), \tag{130}$$

where

$$t(b) = (a+b)^{\frac{2}{\alpha}}\left(a - \frac{2}{\alpha}b\right), \ u(b) = (a-b)^{\frac{2}{\alpha}}\left(a + \frac{2}{\alpha}b\right). \tag{131}$$

It can be readily proved that $t(0) = u(0)$. Through standard calculations, it can also be proved that when $\alpha > 2$ and $b \leq a$, $dt/db(b) \geq du/db(b)$. These results, permit us to conclude that $t(b) \geq u(b)$, or what is the same, the second term in (126) is minimized when $\rho = 0$.

B. *Proof of lemma 3.3*

For any $u \geq 0$, $\tilde{D} > 0$ and $\alpha \geq 4$ define the auxiliary function:

$$\phi(u) := \frac{e^{\tilde{D}^2(1-u^2)} - u^\alpha}{1 - u^\alpha}. \tag{132}$$

Now we show that by replacing

$$g(u) = e^{\tilde{D}^2(1-u^2)} = \sum_{n=0}^{\infty} \frac{\left[\tilde{D}^2(1-u^2)\right]^n}{n!}, \tag{133}$$

in (132) by $g_T(u) := 1 + \tilde{D}^2(1-u^2) + \frac{\tilde{D}^4}{2}(1-u^2)^2$ we obtain a lower bound of (132). When $u \in [0,1]$ the series in (133) is monotonously increasing so in fact $g_T(u) \leq g(u)$. For $u \geq 1$ we have:

$$\frac{dg}{du}(u) = -2\tilde{D}^2 u g(u) \geq -2\tilde{D}^2 u. \tag{134}$$

Now integrate in the interval $[1, u]$ on both sides of (134), add $g(1) = 1$ and apply the fundamental theorem of calculus to show that $g(u) \geq 1 + \tilde{D}^2(1-u^2)$. Repeating the procedure of multiplying by $-2\tilde{D}^2 u$ on both sides and integrating we find that if $u \geq 1$ then $g(u) \leq g_T(u)$. Using this two inequalities we find that for all $u > 0$ we have:

$$\phi(u) \geq \frac{g_T(u) - u^\alpha}{1 - u^\alpha}. \tag{135}$$

$\phi(u)$ and its first derivative have an avoidable singularity at $u = 1$, so using L'Hôpital's rule we compute the first order Taylor expansion of $\phi(u)$ at $u = 1$, $\phi_T(u) = -Mu + N$, where:

$$N = 1 + \tilde{D}^2 + \frac{2\tilde{D}^4}{\alpha}, \qquad M = \tilde{D}^2\left[1 - \frac{2}{\alpha}(1 - \tilde{D}^2)\right], \tag{136}$$

are both positive for $\alpha > 2$ and $\tilde{D} > 0$. Now we show that $u > 0$:

$$\frac{g_T(u) - u^\alpha}{1 - u^\alpha} \geq -Mu + N. \tag{137}$$

This is equivalent to:

$$g_T(u) - u^\alpha - (-Mu + N)(1 - u^\alpha) \begin{cases} \geq 0 & 0 \leq u \leq 1 \\ < 0 & u > 1, \end{cases} \tag{138}$$

which means that we need to study the sign of:

$$p(u,\alpha) = -Mu^{\alpha+1} + (N-1)u^\alpha + \frac{\tilde{D}^4}{2}u^4 - (\tilde{D}^4 + \tilde{D}^2)u^2 + Mu + \tilde{D}^4\left(\frac{1}{2} - \frac{2}{\alpha}\right). \tag{139}$$

First we show that $p(u, \alpha)$ is increasing in α when $0 \leq u \leq 1$ and decreasing in α when $u > 1$; to do this show that $\frac{dp}{d\alpha} > 0$ when $0 \leq u \leq 1$ and $\frac{dp}{d\alpha} < 0$ when $u > 1$. Differentiate p with respect to α to find that:

$$\frac{dp}{d\alpha} = \frac{\tilde{D}^2}{\alpha^2} \left\{ \log(u) u^{\alpha} (1-u) \alpha^2 + 2 \left[(1-u)\tilde{D}^2 + u \right] \left[\left(\log(u^{\alpha}) - 1 \right) u^{\alpha} + 1 \right] \right\}. \tag{140}$$

Since we are interested in the sign of the derivative, we can neglect the factor D^2/α^2. Notice that:

$$(1-u)\tilde{D}^2 + u \begin{cases} \geq u & \text{if } 0 < u \leq 1 \\ < u & \text{if } u > 1, \end{cases} \tag{141}$$

and in addition, $\left[\left(\log(u^{\alpha}) - 1 \right) u^{\alpha} + 1 \right] \geq 0$ when $u > 0$, attaining a minimum at $u = 1$. This can be proven by differentiating:

$$\frac{d \left[\left(\log(u^{\alpha}) - 1 \right) u^{\alpha} + 1 \right]}{du} = \alpha u^{\alpha - 1} \log \left(u^{\alpha} \right), \tag{142}$$

which implies that this function decreases when $0 < u < 1$ and increases when $u > 1$. Using this fact, (141) and defining:

$$f(u, \alpha) = \log(u) u^{\alpha} (1-u) \alpha^2 + 2u \left[\left(\log(u^{\alpha}) - 1 \right) u^{\alpha} + 1 \right] \tag{143}$$

we can bound:

$$\frac{\alpha^2}{\tilde{D}^2} \frac{dp}{d\alpha} \begin{cases} \geq f(u, \alpha) & \text{if } 0 < u \leq 1 \\ < f(u, \alpha) & \text{if } u > 1. \end{cases} \tag{144}$$

When $0 < u \leq 1$ we have that $f(u, \alpha)$ is increasing in α and when $u > 1$ it is decreasing. To see this, differentiate:

$$\frac{df}{d\alpha} = \alpha u^{\alpha} \log(u) \left[2(1-u) + \log(u)(\alpha(1-u) + 2u) \right] \tag{145}$$

an observe that for $u > 0$:

$$2(1-u) + \log(u)(\alpha(1-u) + 2u) \leq 2(1 - u + \log(u)) \leq 0 \tag{146}$$

so the sign of $df/d\alpha$ is the opposite of $\log(u)$ for each pair (u, α). This means that:

$$f(u, \alpha) \begin{cases} \geq f(u, 2) & \text{if } 0 < u \leq 1 \\ < f(u, 2) & \text{if } u > 1. \end{cases} \tag{147}$$

Finally, continue differentiating to show that $f(u,2) \geq 0$ if $0 < u \leq 1$ and $f(u,2) < 0$ when $u > 1$. This means we proved that:

$$\frac{\alpha^2}{\tilde{D}^2}\frac{dp}{d\alpha}\begin{cases} \geq f(u,\alpha) \geq f(u,2) \geq 0 & \text{if } 0 < u \leq 1 \\ < f(u,\alpha) < f(u,2) < 0 & \text{if } u > 1, \end{cases} \tag{148}$$

which concludes the on the monotonicity properties of $p(u,\alpha)$ with respect to α. This implies that it suffices to show that condition (138) is met when $\alpha = 4$. Notice that in that case we have $M = \frac{\tilde{D}}{2}(1+\tilde{D})$, $N = 1 + \tilde{D} + \tilde{D}^2$, which imply that $p(u)$ can be factored as:

$$p(u) = -\frac{1}{2}(\tilde{D} + \tilde{D}^2)u(u-1)^3(u+1). \tag{149}$$

We conclude that (137) is true by observing that (149) satisfies condition (138). To conclude the proof of the lemma use (135), (137), take

$$u = \frac{||r-d||}{D}, \tag{150}$$

$$\tilde{D} = (\lambda_s \delta)^{1/2} D, \tag{151}$$

and multiply both sides of the resulting inequality by $e^{-\lambda_s \delta D^2}$.

C. Complement to the proof of theorem 3.3

To find (55) start by writing:

$$\mathbb{E}_r\left[||r-d||\right] = \int_{\mathbb{R}^2} \frac{1}{2\pi\sigma_{in}^2}||r-d||e^{-\frac{||r||^2}{2\sigma_{in}^2}}dr. \tag{152}$$

Now take $x = r - d$, change to polar coordinates to obtain:

$$\mathbb{E}_r\left[||r-d||\right] = \int_0^\infty \left(\frac{u}{\sigma}\right)^2 e^{-\frac{u^2+D^2}{2\sigma^2}} I_0\left(\frac{Du}{\sigma^2}\right)du \tag{153}$$

$$= \sigma Q_{2,0}\left(\frac{D}{\sigma}, 0\right). \tag{154}$$

In the first step we used the definition of the modified Bessel function. For the actual value of $Q_{2,0}(u,0)$ use (91) from [31] which states that:

$$Q_{2,0}(b,ac) = a\left[\int_c^\infty Q_1(b,ax)dx + cQ_1(b,ac)\right]. \tag{155}$$

Taking $c = 0$:

$$Q_{2,0}(b,0) = a\int_0^\infty Q_1(b,ax)dx. \tag{156}$$

and using (60) of [31] which states that:

$$\int_0^\infty Q_1(b, ax)dx = \frac{\sqrt{2\pi}}{4a} e^{-\frac{b^2}{4}} \left[(b^2+2) I_0\left(\frac{b^2}{4}\right) + b^2 I_1\left(\frac{b^2}{4}\right) \right] \tag{157}$$

we obtain the desired result.

Finally for the upper bound on $\mathbb{E}_r\left[||r-d||\right]$ start from (152), change to polar coordinates and apply lemma D.1. To solve the resulting integral use integration by parts and that:

$$\int_0^{2\pi} \left(2\sin\left(\frac{\theta}{2}\right) - 1 \right) d\theta = 2(4-\pi) \tag{158}$$

$$\int u f(u) du = -\frac{1}{2\pi} e^{-\frac{u^2}{2\sigma^2}} \tag{159}$$

$$\int u^2 f(u) du = -\frac{u}{2\pi} e^{-\frac{u^2}{2\sigma^2}} + \frac{\sigma}{\sqrt{8\pi}} \text{erf}\left(\frac{u}{\sqrt{2}\sigma}\right) \tag{160}$$

(160) is obtained integrating by parts.

APPENDIX C
PROOFS OF SECTION IV

A. *Proof of theorem 4.2*

Rewrite $\mathbb{P}_{\text{out}}(R, p_r)$ in (71) as:

$$\mathbb{P}_{\text{out}}(R,p_r) = \left[1 - \exp\left\{-\lambda_s \delta D^2 \left(1 + \frac{2p_r}{\alpha}\right)\right\}\right] + \frac{p_r D^\alpha}{D^\alpha - \|r-d\|^\alpha} \left[\exp\left\{-\lambda_s \delta D^2 \left(1 + \frac{2p_r}{\alpha}\right)\right\} - \exp\left\{-\lambda_s \delta \|r-d\|^2 \left(1 + \frac{2p_r}{\alpha}\right)\right\}\right]. \tag{161}$$

It is immediate to show that the first term in (161) is concave in p_r. Hence, only the concavity of the second term is to be proved:

$$h(p_r) = \frac{p_r D^\alpha}{D^\alpha - \|r-d\|^\alpha} \left[\exp\left\{-\lambda_s \delta D^2 \left(1 + \frac{2p_r}{\alpha}\right)\right\} - \exp\left\{-\lambda_s \delta \|r-d\|^2 \left(1 + \frac{2p_r}{\alpha}\right)\right\}\right]. \tag{162}$$

The second derivative $dh^2/dp_r^2(p_r)$ of $h(p_r)$ can be written as:

$$\frac{d^2 h}{dp_r^2}(p_r) = \frac{4\lambda_s \delta D^\alpha}{\alpha(D^\alpha - \|r-d\|^\alpha)} \times \left[\left(1 - \frac{p_r}{\alpha}\lambda_s \delta \|r-d\|^2\right) \|r-d\|^2 \exp\left\{-\lambda_s \delta \|r-d\|^2 \left(1 + \frac{2p_r}{\alpha}\right)\right\} - \left(1 - \frac{p_r}{\alpha}\lambda_s \delta D^2\right) D^2 \exp\left\{-\lambda_s \delta D^2 \left(1 + \frac{2p_r}{\alpha}\right)\right\}\right]. \tag{163}$$

Let us analyze the value of the function $\frac{d^2h}{dp_r^2}(p_r)$ when $p_r = 0$. We obtain:

$$\frac{d^2h}{dp_r^2}(0) \;=\; \frac{4\lambda_s\delta D^\alpha}{\alpha(D^\alpha - \|r-d\|^\alpha)}\left[\|r-d\|^2\exp\left\{-\lambda_s\delta\|r-d\|^2\right\} - D^2\exp\left\{-\lambda_s\delta D^2\right\}\right]. \quad (164)$$

Consider the case $u \equiv \frac{\|r-d\|^2}{D^2} < 1$. It is easy to verify that $d^2h/du^2(0) < 0$ if:

$$\lambda_s\delta D^2 \le -\frac{\ln u}{1-u}, \quad u \in [0,1]. \quad (165)$$

It can be shown that $-\frac{\ln u}{1-u}$ is decreasing in $[0,1]$ with

$$\lim_{u\to 0} -\frac{\ln u}{1-u} = +\infty, \quad \lim_{u\to 1} -\frac{\ln u}{1-u} = 1. \quad (166)$$

This means that when $u \in [0,1]$, $\frac{d^2h}{dp_r^2}(0) < 0$ if $\lambda_s\delta D^2 < 1$. The case $u > 1$ can be handled in the same way to obtain that when $u > 1$, $\frac{d^2h}{dp_r^2}(0) < 0$ if $\lambda_s\delta\|r-d\|^2 < 1$. Finally, the case $u = 1$ can be treated with appropriate continuity arguments. Summarizing, we have that $\frac{d^2h}{dp_r^2}(0) < 0$ if

$$\lambda_s\delta\max\left\{D^2, \|r-d\|^2\right\} < 1. \quad (167)$$

As $h(p_r)$ is continuous function, the fact that $\frac{d^2h}{dp_r^2}(0) < 0$ permit us to assure that there exists an interval $[0,\beta]$ with $\beta > 0$ where $\frac{d^2h}{dp_r^2}(p_r) \le 0$ if $p_r \in [0,\beta]$. In order to guarantee the concavity of $h(p_r)$ for $p_r \in [0,1]$ we need to find conditions for $\beta \ge 1$. We need to consider (163). Consider again $u < 1$. In order to find the points p_r^* where $\frac{d^2h}{dp_r^2}(p_r^*) = 0$ we need to solve:

$$g(p_r^*) = \exp\left\{-\lambda_s\delta D^2\left(1 + \frac{2p_r^*}{\alpha}\right)(1-u)\right\}, \quad (168)$$

where

$$g(p_r) = u\frac{1 - \frac{p_r}{\alpha}\lambda_s\delta\|r-d\|^2}{1 - \frac{p_r}{\alpha}\lambda_s\delta D^2}. \quad (169)$$

It can be easily shown that $g(p_r)$ is an increasing function of p_r is $u < 1$. Note that $g(p_r)$ has a vertical asymptote at $\frac{\alpha}{\lambda_s\delta D^2}$ and zero at $\frac{\alpha}{\lambda_s\delta\|r-d\|^2}$. As $e^{-\lambda_s\delta D^2\left(1+\frac{2p_r^*}{\alpha}\right)(1-u)}$ is decreasing in p_r when $u < 1$, it is not difficult to see that there exist only two values p_r^* such that (168) is satisfied, and such points satisfy:

$$p_r^{(1)} < \frac{\alpha}{\lambda_s\delta D^2}, \quad p_r^{(2)} > \frac{\alpha}{\lambda_s\delta\|r-d\|^2}. \quad (170)$$

From the condition (167) and the fact that $\alpha > 2$ it is clear that $p_r^{(2)}$ is of not use for us. We just need to concentrate on proving that $p_r^{(1)} > 1$. As $g(p_r)$ is an increasing function of p_r, we just need to prove:

$$g(1) \le \exp\left\{-\lambda_s\delta D^2\left(1 + \frac{2}{\alpha}\right)(1-u)\right\}. \quad (171)$$

Consider the existence of a value $K > 1$ such that:

$$\frac{1 - \frac{\lambda_s \delta \|r-d\|^2}{\alpha}}{1 - \frac{\lambda_s \delta D^2}{\alpha}} \leq K. \tag{172}$$

It is straightforward to obtain:

$$\lambda_s \delta D^2 \leq \frac{\alpha(K-1)}{K-u}. \tag{173}$$

From (171) and (172) we obtain also the following condition:

$$\lambda_s \delta D^2 \leq -\frac{\ln(Ku)}{(1+\frac{2}{\alpha})(1-u)}, \quad K < \frac{1}{u}. \tag{174}$$

Combining (173) and (174) we obtain the following condition which guarantee us that (171) be satisfied:

$$\lambda_s \delta D^2 \leq \beta^*, \tag{175}$$

where β^* is defined as $\beta^* = \inf_{u<1} \beta(u)$ and

$$\beta(u) = \max_{1 \leq K \leq \frac{1}{u}} \min \left\{ \frac{\alpha(K-1)}{K-u}, -\frac{\ln(Ku)}{(1+\frac{2}{\alpha})(1-u)} \right\}. \tag{176}$$

Notice that β^* depends only on α and can be optimized numerically. This numerical optimization suggests that the value of $\beta^* = 0.38$ for $\alpha = 2$ is a lower bound for all the values of β^* for other values of $\alpha > 2$. In this way the can conclude that $h(p_r)$ is concave as a function of p_r if $\lambda_s \delta D^2 \leq 0.38$. The case $u > 1$ is treated in the same way. The conclusion of the lemma follows straightforwardly.

B. *Proof of lemma 4.2*

For the first item consider that:

$$\frac{d^2 m}{dp_r^2}(p_r) = \frac{2\lambda_s \delta}{\alpha} \left[\|r\|^2 - D^\alpha \frac{D^2 - \|r-d\|^2}{D^\alpha - \|r-d\|^\alpha} \right]. \tag{177}$$

In order to be negative we need:

$$\|r\|^2 \leq D^\alpha \frac{D^2 - \|r-d\|^2}{D^\alpha - \|r-d\|^\alpha}. \tag{178}$$

Defining $x = \frac{\|r-d\|^2}{D^2}$, we obtain that in order to $\frac{d^2 m}{dp_r^2}(p_r)$ to be negative the following two conditions have to be satisfied in $r \in \mathbb{R}^2$:

$$\|r-d\|^2 = xD^2, \quad \|r\|^2 \leq D^2 \frac{1-x}{1-x^{\frac{\alpha}{2}}}, \quad \text{for some } x > 0. \tag{179}$$

This conclude the proof of the first item. For the second we have:

$$m(0) = \lambda_s \delta D^2, \tag{180}$$

$$m(1) = \lambda_s \delta \left(1 + \frac{2}{\alpha}\right) \left[D^2 + \|r\|^2 - D^\alpha \frac{D^2 - \|r-d\|^2}{D^\alpha - \|r-d\|^\alpha}\right]. \tag{181}$$

In order to see when $p_r = 1$ is optimal we need to consider $m(0) - m(p_r) > 0$. It is easy to see that this happens when:

$$D^2 \frac{1-x}{1-x^{\frac{\alpha}{2}}} \left(1 + \frac{2}{\alpha}\right) \geq \frac{2D^2}{\alpha} + \|r\|^2 \left(1 + \frac{2}{\alpha}\right), \quad \|r-d\|^2 = xD^2, \quad \text{for some } x > 0. \tag{182}$$

Rearranging terms and using terms, we have:

$$\|r\|^2 \leq D^2 \left[\frac{1-x}{1-x^{\frac{2}{\alpha}}} - \frac{2}{\alpha+2}\right], \ \|r-d\|^2 = xD^2, \quad \text{for some } x > 0. \tag{183}$$

This concludes the proof of the lemma.

C. *Proof of theorem 4.6*

First we have to evaluate the OP conditioned on the position of the NN of the source at the origin, that is, evaluate (90) without the expectation with respect to r. Notice that in that case the expression of the OP in terms of the Laplace transforms of the interference RVs is the same as in the fixed relay case, and therefore (81), which we repeat here for comfort, is also valid:

$$\mathbb{P}_{\text{out,mix}}(R, p_r) = (1-p_r)\left[1 - \mathcal{L}_{I_d}\left(T/l_{sd}\right)\right] + p_r \left[1 - \frac{D^\alpha \mathcal{L}_{I_d}\left(T/l_{rd}\right)}{D^\alpha - \|r-d\|^\alpha} + \frac{||r-d||^\alpha \mathcal{L}_{I_d}\left(T/l_{sd}\right)}{D^\alpha - \|r-d\|^\alpha}\right] + p_r\left[1 - \mathcal{L}_{I_r}\left(T/l_{sr}\right)\right]. \tag{184}$$

When the Laplace transform is evaluated we have to take into account that the nearest neighbor of each cluster is also random. However considering the approximated interference RVs (64) and (63) and lemma A.2 in appendix A the Laplace transform in this case are also:

$$\mathcal{L}_{I_d}(\omega_1) = \exp\left\{-\lambda_s C \omega_1^{2/\alpha} \left(1 + \frac{2p_r}{\alpha}\right)\right\}, \tag{185}$$

where δ is given by (36) and C is defined as in (35). Now we have to average (184) with respect to the position of the origin's nearest neighbor. It is clear that the first term of (36) is not dependent on r. In the second term taking $u = ||r-d||/D$ we can write:

$$\frac{D^\alpha \mathcal{L}_{I_d}\left(T/l_{rd}\right)}{D^\alpha - \|r-d\|^\alpha} - \frac{||r-d||^\alpha \mathcal{L}_{I_d}\left(T/l_{sd}\right)}{D^\alpha - \|r-d\|^\alpha} = e^{-\lambda_s \Delta D^2} \left(\frac{e^{\lambda_s \Delta D^2(1-u^2)} - u^\alpha}{1-u^\alpha}\right). \tag{186}$$

Using first that $e^{\lambda_s \Delta D^2(1-u^2)} = 1+\lambda_s \Delta D^2(1-u^2)+O((\lambda_s \Delta D^2(1-u^2))^2)$ as $(\lambda_s \Delta D^2(1-u^2)) \to 0$ and then that for $u > 0$:

$$\frac{1-u^2}{1-u^\alpha} \geq 1 + \left(\frac{2}{\alpha} - 1\right) u, \tag{187}$$

we find that:

$$\left(\frac{e^{\lambda_s \Delta D^2(1-u^2)} - u^\alpha}{1-u^\alpha}\right) \geq 1 + \lambda_s \Delta D^2 \left[1 + \left(\frac{2}{\alpha} - 1\right) u\right] + O\left((\lambda_s \Delta D^2)^2 \frac{(1-u^2)^2}{|1-u^\alpha|}\right). \tag{188}$$

as $\lambda_s \Delta D^2(1-u^2) \to 0$. Taking expectation with respect to $u = ||r-d||/D$ we obtain:

$$\mathbb{E}_r \left[\frac{e^{\lambda_s \Delta D^2(1-u^2)} - u^\alpha}{1-u^\alpha}\right] \geq 1 + \lambda_s \Delta D^2 \left[1 + \left(\frac{2}{\alpha} - 1\right) \mathbb{E}_r \left[u\right]\right] + O\left((\lambda_s \Delta D^2)^2\right). \tag{189}$$

The third term can be evaluated in a straightforward manner as in lemma 3.3 to yield:

$$\mathbb{E}_r \left[\mathcal{L}_{I_r}\left(T/l_{sr}\right)\right] = \frac{\lambda_s \Delta}{\pi \lambda_{in} + \lambda_s \Delta}. \tag{190}$$

D. *Proof of theorem 4.7*

Define $\nu(p_r) := \lambda_s \Delta(p_r) D^2$ to write the OP as:

$$\begin{aligned} \mathbb{P}_{\text{out,mix}}(R) \leq & \left[1 - \frac{\alpha(\nu(p_r) - \nu(0))}{2\nu(0)}\right]\left[1 - e^{-\nu(p_r)}\right] + \frac{\alpha(\nu(p_r) - \nu(0))}{2\nu(0)} \left[1 + \frac{\nu(p_r)}{\pi \lambda_{in} D^2 + \nu(p_r)}\right] \\ & - \frac{\alpha(\nu(p_r) - \nu(0))}{2\nu(0)} e^{-\nu(p_r)} \left[1 + \nu(p_r)\left(1 + \frac{2-\alpha}{\alpha D} \mathbb{E}_r\left[||r-d||\right]\right)\right]. \end{aligned} \tag{191}$$

It is clear that analyzing the concavity with respect to $\nu(p_r)$ is the same as analyzing it with respect to p_r. Our goal is to differentiate twice with respect to ν to find conditions for the OP to be a concave function of p_r. We have found the exact value of $\mathbb{E}_r\left[||r-d||\right]$ in terms of the Nuttall $Q_{2,0}$ function (55) but this expression is not easy to handle. For this reason we would like to use a suitable bound for $\mathbb{E}_r\left[||r-d||\right]$, that is, we have to differentiate twice and then bound the expectation accordingly. For this reason we first find conditions that guarantee that an upper bound on the expectation will yield an upper bound of the second derivative of the OP. Let us define the auxiliary function:

$$\phi(\nu(p_r), u) := -e^{-\nu(p_r)}(\nu(p_r) - \nu(0))\left(1 + \nu(p_r) u\right) \tag{192}$$

which is similar to the third term of (191), and study its behaviour with u. Differentiate $\phi(\nu, u)$ twice with respect to ν and then with respect to u to find:

$$\frac{d}{du}\left(\frac{d^2\phi}{d\nu^2}\right) = e^{-\nu}\left[-\nu^2 + (4 + \nu(0))\nu - 2(1 + \nu(0))\right], \tag{193}$$

where $\lambda_s\delta = \nu(0) \leq \nu(p_r) \leq \nu(1) = \lambda_s\delta\left(1+\frac{2}{\alpha}\right)$. Notice that the derivative does not depend on u. It suffices to study the values of the function at the borders of the interval of ν to see how the derivative behaves with u. When $p_r = 0$ we have $\nu = \lambda_s\delta$ and we find that the derivative is negative as long as $\lambda_s\delta D^2 \leq 1$. When $p_r = 1$ then $\nu = \lambda_s\delta\left(1+\frac{2}{\alpha}\right)$ and we find that the derivative is negative if:

$$\nu(0) = \lambda_s\delta D^2 \leq \frac{4+\alpha\left[1-\left(1+4/\alpha+8/\alpha^2\right)^{1/2}\right]}{2(1+2/\alpha)}. \tag{194}$$

It is easy to show that the expression on the right is increasing in α which means that the function attains a minimum at $\alpha = 2$. To do this show that the numerator is increasing in α and the denominator is decreasing in α, and that they are both positive. Therefore setting $\alpha = 2$ in (194) gives a sufficient condition for the second derivative to be negative in ν in all the interval:

$$\nu(0) = \lambda_s\delta D^2 \leq \frac{3-\sqrt{5}}{2} \approx 0.38. \tag{195}$$

Therefore if condition (195) is met it is easy to show that since $\alpha > 2$ we can differentiate (191) twice with respect to ν, then use (56) and study the conditions for the concavity of the OP with respect to p_r. Now do this and reorder the terms to find:

$$\frac{d^2\mathbb{P}_{\text{out,mix}}}{d\nu^2} \leq \frac{\alpha\pi\lambda_{in}D^2}{\nu(0)}\frac{\nu(0)+\pi\lambda_{in}D^2}{(\nu(p_r)+\pi\lambda_{in}D^2)^3} - \frac{e^{-\nu(p_r)}}{\nu(0)}\left[A_2(s)\nu(p_r)^2 + A_1(s)\nu(p_r) + A_0(s)\right] \tag{196}$$

with:

$$A_2(s) = 1 + \left(1-\frac{\alpha}{2}\right)\gamma(s)s, \tag{197}$$

$$A_1(s) = -\left[1+\left(1-\frac{\alpha}{2}\right)\gamma(s)s\right]\left[4+\nu(0)\right], \tag{198}$$

$$A_0(s) = 2\left[1+\left(1-\frac{\alpha}{2}\right)\gamma(s)s\right]\left[1+\nu(0)\right] + \nu(0). \tag{199}$$

and $s = \sigma_{in}/D$. Now apply the bound:

$$\frac{\alpha\pi\lambda_{in}D^2}{\nu(0)}\frac{\nu(0)+\pi\lambda_{in}D^2}{(\nu(p_r)+\pi\lambda_{in}D^2)^3} \leq \frac{\alpha}{\nu(0)\pi\lambda_{in}D^2} = \frac{2\alpha s^2}{\nu(0)}, \tag{200}$$

and use that $2\sigma_{in}^2 = (\pi\lambda_{in})^{-1}$ which is straightforward from the fact that $\nu(p_r) \geq \nu(0) = \lambda_s\delta D^2 > 0$. Then factor out the term $e^{-\nu(p_r)}/\nu(0)$ to obtain:

$$\frac{d^2\mathbb{P}_{\text{out,mix}}}{d\nu^2(p_r)} \leq \frac{e^{-\nu(p_r)}}{\nu(0)}\left[2\alpha e^{\nu(p_r)}s^2 - A_0(s) - A_1(s)\nu(p_r) - A_2(s)\nu^2(p_r)\right]. \tag{201}$$

Now using that $\alpha > 2$ we upper bound:

$$-A_1 = \left[1 + \left(1 - \frac{\alpha}{2}\right)\gamma s\right][4 + \nu(0)] \leq 4 + \nu(0). \tag{202}$$

Now using (202) in (201) we find that we can find sufficient conditions on s for the concavity of the OP by studying the sign of the function:

$$p_c(\nu(p_r), s) := 2\alpha e^{\nu(p_r)} s^2 - A_0 + (4 + \nu(0))\nu(p_r) - A_2\nu^2(p_r). \tag{203}$$

To do this, differentiate (203) with respect to ν to find that $p_c(\nu, s)$ is increasing in ν for all $s > 0$ whenever (195) is met. This means that a sufficient condition for the concavity of the OP is that $p_c(\nu(1), s) < 0$. Now we can rewrite p_c explicitly as a function of s, evaluate at $\nu(1)$ and remembering that $\nu(1) = \nu(0)(1 + 2/\alpha)$, upper bound it:

$$p_c(\nu(1), s) = 2\alpha e^{\nu(1)} s^2 + (\alpha - 2)\left(\frac{\nu^2(1)}{2} + \nu(0) + 1\right)\gamma(s)s + C_0(\nu(1)), \tag{204}$$

$$\leq 2\alpha e^{\nu(1)} s^2 + (\alpha - 2)e^{\nu(1)}\gamma(s)s + C_0(\nu(1)), \tag{205}$$

where $C_0(\nu(1)) = -\nu^2(1) + (4 + \nu(0))\nu(1) - (2 + 3\nu(0))$. First notice that $C_0(\nu(1)) < 0$ for $\alpha > 2$ and as long as $\nu(0) \leq 1/2$. To show this notice that $C_0(\nu(1))$ can be written as:

$$C_0(\nu(1)) = \frac{1}{\alpha^2}\left[(\nu(0) - 2)\alpha^2 + 2\nu(0)(4 - \nu(0))\alpha - 4\nu(0)^2\right], \tag{206}$$

which we want to prove is negative for $\alpha > 2$. Since $\nu(0)$ satisfies (195) we deduce that this function has a single positive root in α, which is:

$$\alpha_0 = \nu(0)\left[\frac{4 - \nu(0)}{2 - \nu(0)} + \frac{\sqrt{\nu(0)^2 - 4\nu(0) + 8}}{2 - \nu(0)}\right], \tag{207}$$

and we want to prove that $\alpha_0 \leq 2$. Differentiate the function between brackets in (207) to show that it is increasing in $\nu(0)$, and therefore, α_0 is increasing in $\nu(0)$. Taking $v = 1/2$ shows that in that case $\alpha_0 = 2$ so we get that $C_0(\nu(1)) < 0$ when (195) is met. Since the other two terms of (205) are positive for all $\nu > 0$ it is clear that there will be an upper bound on s for the OP to be a concave function of p_r. Finally, a sufficient condition for the OP to be a concave function is finding the positive root in s of the upper bound in (205):

$$e^{\lambda_s \Delta D^2}\left(2\alpha s^2 + (\alpha - 2)\gamma(s)s\right) - (\lambda_s \Delta D^2)^2 + (4 + \lambda_s \delta D^2)\lambda_s \Delta D^2 - (2 + 3\lambda_s \delta D^2) = 0. \tag{208}$$

Notice that since $\gamma(s)$ is transcendental function of s we cannot find the roots explicitly. However we can upper bound the value of $\gamma(s)$ in the following way. For each α it is clear that the maximum value of s is attained when $\lambda_s\delta D^2 = 0$, and in that case (208) becomes:

$$2\alpha s^2 + (\alpha - 2)\gamma(s)s - 2 = 0. \tag{209}$$

It is easy to show that the roots in terms of s are decreasing with α, and therefore for $\alpha = 2$ the root is the largest over all $\alpha > 2$ and $\lambda_s\delta D^2$. In that case we have that (209) is independent of $\gamma(s)$ and the value of s is $s = 2^{-1/2}$. Therefore, solving numerically we find that for all $\alpha > 2$ and $\lambda_s\delta D^2 > 0$ we can take $\gamma(s) \leq 1/2$. In that case (208) is a second degree polynomial in s and its roots can be found in closed form. To find (94) take the positive root of this polynomial and find a lower bound by discarding the square of the linear coefficient inside the square root of the expression.

E. Proof of theorem 4.8

We want to find conditions on σ_{in} that guarantee that $\mathbb{P}_{\text{out,DF}}(p_r = 1) \leq \mathbb{P}_{\text{out,DF}}(p_r = 0)$. Setting $p_r = 0$ and $p_r = 1$ we can write:

$$\mathbb{P}_{\text{out,mix}}(p_r = 0) = 1 - e^{-\lambda_s\delta D^2} \tag{210}$$

$$\mathbb{P}_{\text{out,mix}}(p_r = 1) = 1 + \frac{\lambda_s\Delta}{\lambda_s\Delta + \pi\lambda_{in}} - \left[1 + \lambda_s\Delta D^2\left(1 + \frac{2-\alpha}{\alpha D}\mathbb{E}_r\left[||r - d||\right]\right)\right]e^{-\lambda_s\Delta D^2}. \tag{211}$$

An explicit for the values for which $\mathbb{P}_{\text{out,mix}}(p_r = 1) \leq \mathbb{P}_{\text{out,mix}}(p_r = 0)$ cannot be found. Therefore we find a sufficient condition by bounding (210) and (211). First we use (56) and:

$$\frac{\lambda_s\Delta}{\lambda_s\Delta + \pi\lambda_{in}} \leq \frac{\lambda_s\Delta}{\pi\lambda_{in}} = 2\lambda_s\Delta\sigma_{in}^2 \tag{212}$$

in (211) to upper bound:

$$\mathbb{P}_{\text{out,mix}}(p_r = 1) \leq 1 + 2\lambda_s\Delta\sigma_{in}^2 - \left\{1 + \lambda_s\Delta D^2\left[1 + \frac{2-\alpha}{\alpha}\left(1 + \gamma(s)\frac{\sigma_{in}}{D}\right)\right]\right\}e^{-\lambda_s\Delta D^2}. \tag{213}$$

Therefore a sufficient condition for $p_r = 1$ to be optimal is obtained by asking that the right side of (213) to be smaller than (210). Writing this condition an reordering yields that $p_r = 1$ will be optimal when:

$$2\lambda_s\Delta D^2 s^2 + \lambda_s\Delta D^2\left(1 - \frac{2}{\alpha}\right)e^{-\lambda_s\Delta D^2}\gamma(s)s + e^{-\lambda_s\delta D^2} - e^{-\lambda_s\Delta D^2}\left(1 + \frac{2}{\alpha}\lambda_s\Delta D^2\right) \leq 0, \tag{214}$$

with $s = \sigma_{in}/D$. Now upper bound the terms that do not depend on s as:

$$e^{-\lambda_s \delta D^2} - e^{-\lambda_s \Delta D^2}\left(1 + \frac{2}{\alpha}\lambda_s \Delta D^2\right) \leq \frac{4\lambda_s \delta D^2}{\alpha^2} e^{-\lambda_s \Delta D^2}\left(\frac{2}{3}\lambda_s \delta D^2 - 1\right) \tag{215}$$

using that when condition (195) is met then $e^u \leq 1 + u + \frac{2}{3}u^2$. This turns the sufficient condition (214) into:

$$2\alpha\left(\alpha+2\right)e^{\lambda_s \Delta D^2} s^2 + \left(\alpha^2 - 4\right)\gamma(s)s + 4\left(\frac{2}{3}\lambda_s \delta D^2 - 1\right) \leq 0, \tag{216}$$

Again by setting equality in (216) we can find the maximum value of s so that $p_r = 1$ is optimal. However this equation does not have an explicit solution and must be solved numerically. To work around this problem and obtain a closed form expression, observe that for each α as $\lambda_s \Delta D^2$ grows the maximum value of s that satisfies (216) decreases so that the maximum value of s is attained when $\lambda_s \delta D^2 = 0$, which is also intuitively right. Therefore setting $\lambda_s \delta D^2 = 0$ in (216) yields an upper bound for the condition for each α:

$$2\alpha\left(\alpha+2\right)s^2 + \left(\alpha^2 - 4\right)\gamma(s)s - 4 \leq 0. \tag{217}$$

It is easy to see that as α grows the maximum value of s that satisfies the equality in (217) decreases and therefore setting $\alpha = 2$ yields an overall upper bound for s for all $\alpha > 2$ and $\lambda_s \delta D^2 > 0$. Doing this we obtain that $s \leq 1/2$ which yields $\gamma(s) \leq 2/5$. Upper bounding γ in (216) turns the condition the equation into a second degree polynomial in s that can be solved in closed form. Once more to find (96) take the positive root of this polynomial and lower bound it by neglecting the square of the linear coefficient inside the square root of the expression.

APPENDIX D
AUXILIARY RESULTS

A. *CCDF of a quadratic form of complex circularly symmetric Gaussian RVs*

We want to calculate the distribution of: $V := |h_{sd}|^2 l_{sd} + |h_{rd}|^2 l_{rd} + 2\sqrt{l_{sd} l_{rd}}\Re(\rho h_{sd} h_{rd}^*)$, which can be factorized as:

$$V = q^H \begin{bmatrix} l_{sd} & \sqrt{l_{sd} l_{rd}}\rho \\ \sqrt{l_{sd} l_{rd}}\rho^* & l_{rd} \end{bmatrix} q := q^H Q q, \tag{218}$$

where $q = \begin{bmatrix} h_{sd} & h_{rd} \end{bmatrix}^T$, is a zero-mean complex circularly symmetric Gaussian vector with identity covariance matrix. Since Q is positive definite we can diagonalize it to obtain $V =$

$q^H P D P^H q$, where P is a unitary matrix. Defining $w := \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T = Pq$ we have that $V = w^H D w = D_{1,1}|w_1|^2 + D_{2,2}|w_2|^2$, where $D_{1,1}$ and $D_{2,2}$ are the elements on the main diagonal of D. Since P is unitary, $w = P^H q$ has the same distribution as q so $\{|w_1|^2, |w_2|^2\}$ are i.i.d. exponential RVs with unit mean. This means that $D_{1,1}|w_1|^2$ and $D_{2,2}|w_2|^2$ are independent exponential RVs with means $D_{1,1}$ and $D_{2,2}$ respectively, where:

$$D_{1,1} = \frac{1}{2}\left[(l_{sd} + l_{rd}) + \left((l_{sd} - l_{rd})^2 + 4 l_{sd} l_{rd} |\rho|^2\right)^{1/2}\right] \tag{219}$$

$$D_{2,2} = \frac{1}{2}\left[(l_{sd} + l_{rd}) - \left((l_{sd} - l_{rd})^2 + 4 l_{sd} l_{rd} |\rho|^2\right)^{1/2}\right], \tag{220}$$

are the eigenvalues of Q which are on the main diagonal of Q. In Appendix D-B taking $V = X + Y$, $\mu_1 = D_{1,1}$ and $\mu_2 = D_{2,2}$ we obtain:

$$\bar{F}_V(z) = \begin{cases} 1 - \frac{\mu_2 e^{-z/\mu_2} - \mu_1 e^{z/\mu_1}}{\mu_2 - \mu_1} & \mu_1 \neq \mu_2 \\ 1 - (1 + z/\mu_1) e^{-z/\mu_1} & \mu_1 = \mu_2. \end{cases} \tag{221}$$

Observe that $\mu_1 = \mu_2$ if and only if $l_{sd} = l_{rd}$ and $\rho = 0$.

B. CDF of the sum of independent exponential RVs

Let $\{X, Y\}$ be independent exponential RVs with means μ_1 and μ_2 respectively. The distribution of their sum is:

- If $\mu_1 = \mu_2$:

$$F_{X+Y}(z) = \int_0^z f_X(x) \left(\int_0^{z-x} f_Y(y) dy\right) dx \tag{222}$$

$$= 1 - (1 + \mu_1^{-1} z) e^{-z/\mu_1}. \tag{223}$$

- If $\mu_1 \neq \mu_2$:

$$F_{X+Y}(z) = \int_0^z f_X(x) \left(\int_0^{z-x} f_Y(y) dy\right) dx \tag{224}$$

$$= 1 - \frac{\mu_2 e^{-z/\mu_2} - \mu_1 e^{-z/\mu_1}}{\mu_2 - \mu_1}. \tag{225}$$

Finally:

$$F_{X+Y}(z) = \begin{cases} 1 - \frac{\mu_2 e^{-z/\mu_2} - \mu_1 e^{-z/\mu_1}}{\mu_2 - \mu_1} & \mu_1 \neq \mu_2 \\ 1 - (1 + z/\mu_1) e^{-z/\mu_1} & \mu_1 = \mu_2. \end{cases} \tag{226}$$

C. An additional bound

Lemma D.1: Let r and d be fixed vector in $\mathbb{R}^2$ and let $\theta \in [0, 2\pi)$ be the angle between r and d. Then the following bound is valid:

$$||d - r|| \leq |||d|| - ||r||| + 2 \min (||d||, ||r||) \, | \sin (\theta/2) \, |. \tag{227}$$

Proof: Write:

$$r - d = u_1 + v_1 \tag{228}$$

$$r - d = u_2 + v_2, \tag{229}$$

where:

$$u_1 = ||r|| \left(\frac{r}{||r||} - \frac{d}{||d||} \right) \qquad v_1 = \frac{d}{||d||} (||r|| - ||d||) \tag{230}$$

$$u_2 = ||d|| \left(\frac{r}{||r||} - \frac{d}{||d||} \right) \qquad v_2 = \frac{r}{||r||} (||r|| - ||d||) \, . \tag{231}$$

Use the Pythagorean theorem to show that:

$$||u_1|| = 2||r|| \sin(\theta/2) \tag{232}$$

$$||u_2|| = 2||d|| \sin(\theta/2) \tag{233}$$

$$||v_1|| = ||v_2|| = |\|r\| - \|d\|| \, . \tag{234}$$

Finally apply the triangle inequality to (228) and (229) to find:

$$||r - d|| \leq |\|r\| - \|d\|| + 2||r|| \sin(\theta/2) \tag{235}$$

$$||r - d|| \leq |\|r\| - \|d\|| + 2||d|| \sin(\theta/2). \tag{236}$$

To conclude we can take the minimum between $||r||$ and $||d||$ from the previous equations. ■

References

[1] G. Kramer, M. Gastpar, and P. Gupta, “Cooperative strategies and capacity theorems for relay networks,” *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.

[2] T. Cover and A. E. Gamal, “Capacity theorems for the relay channel,” *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572–584, Sep. 1979.

[3] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, “An aloha protocol for multihop mobile wireless networks,” *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 421–436, Feb. 2006.

[4] S. Weber, J. G. Andrews, and N. Jindal, “An overview of the transmission capacity of wireless networks,” *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3593–3604, Dec. 2010.

[5] F. Baccelli, A. El Gamal, and D. N. Tse, “Interference networks with Point-to-Point codes,” *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2582–2596, May 2011.

[6] J. N. Laneman, D. N. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062– 3080, Dec. 2004.

[7] A. Host-Madsen and J. Zhang, “Capacity bounds and power allocation for wireless relay channels,” *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2020–2040, Jun. 2005.

[8] M. Katz and S. Shamai, “Cooperative schemes for a source and an occasional nearby relay in wireless networks,” *IEEE Trans. Inf. Theory*, vol. 55, no. 11, pp. 5138–5160, Nov. 2009.

[9] P. Gupta and P. R. Kumar, “The capacity of wireless networks,” *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[10] ——, “Towards an information theory of large networks: an achievable rate region,” *IEEE Trans. Inf. Theory*, vol. 49, no. 8, pp. 1877– 1894, Aug. 2003.

[11] L. Xie and P. R. Kumar, “A network information theory for wireless communication: scaling laws and optimal operation,” *IEEE Trans. Inf. Theory*, vol. 50, no. 5, pp. 748– 767, May 2004.

[12] F. Xue, L. L. Xie, and P. R. Kumar, “The transport capacity of wireless networks over fading channels,” *IEEE Trans. Inf. Theory*, vol. 51, no. 3, pp. 834–847, Mar. 2005.

[13] F. Baccelli and B. Blaszczyszyn, *Stochastic Geometry and Wireless Networks*. NoW Publishers, 2009.

[14] D. J. Daley and D. Vere-Jones, *An introduction to the theory of point processes. Vol. II*, 2nd ed. New York: Springer, 2008.

[15] R. K. Ganti and M. Haenggi, “Interference and outage in clustered wireless ad hoc networks,” *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 4067–4086, Sep. 2009.

[16] S. P. Weber, X. Yang, J. G. Andrews, and G. de Veciana, “Transmission capacity of wireless ad hoc networks with outage constraints,” *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4091–4102, Dec. 2005.

[17] A. M. Hunter, J. Andrews, and S. Weber, “Transmission capacity of ad hoc networks with spatial diversity,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5058–5071, Dec. 2008.

[18] N. Jindal, S. Weber, and J. G. Andrews, “Fractional power control for decentralized wireless networks,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5482–5492, Dec. 2008.

[19] B. Picinbono, “On circularity,” *IEEE Trans. Signal Process.*, vol. 42, no. 12, pp. 3473–3482, Dec. 1994.

[20] H. Inaltekin, S. B. Wicker, M. Chiang, and H. V. Poor, “On unbounded path-loss models: effects of singularity on wireless network performance,” *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1078–1092, Sep. 2009.

[21] M. Z. Win, P. C. Pinto, and L. A. Shepp, “A mathematical theory of network interference and its applications,” *Proceedings of the IEEE*, vol. 97, no. 2, pp. 205–230, Feb. 2009.

[22] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, “Stochastic geometry and random graphs for the analysis and design of wireless networks,” *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029–1046, Sep. 2009.

[23] P. Billingsley, *Probability and Measure*, 2nd ed. Wiley-Interscience, 1986.

[24] A. Behboodi and P. Piantanida, “On the asymptotic error probability of composite relay channels,” in *Proc. IEEE Int. Symp. Information Theory*, Aug. 2011, pp. 1524–1528.

[25] L. Zheng and D. N. Tse, “Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels,” *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1073– 1096, May 2003.

[26] A. Carleial, “Multiple-access channels with different generalized feedback signals,” *IEEE Trans. Inf. Theory*, vol. 28, no. 6, pp. 841–850, Nov. 1982.

[27] F. Willems and E. van der Meulen, “The discrete memoryless multiple-access channel with cribbing encoders,” *IEEE Trans. Inf. Theory*, vol. 31, no. 3, pp. 313–327, May 1985.

[28] C. M. Zeng, F. Kuhlmann, and A. Buzo, “Achievability proof of some multiuser channel coding theorems using backward decoding,” *IEEE Trans. Inf. Theory*, vol. 35, no. 6, pp. 1160–1165, Nov. 1989.

[29] L. Zhang, J. Jiang, A. J. Goldsmith, and S. Cui, “Study of gaussian relay channels with correlated noises,” *IEEE Trans. Commun.*, vol. 59, no. 3, pp. 863–876, Mar. 2011.

[30] A. Altieri, L. Rey Vega, C. G. Galarza, and P. Piantanida, “Cooperative strategies for interference-limited wireless networks,” in *Proc. IEEE Int. Symp. Information Theory*, Aug. 2011, pp. 1623–1627.

[31] H. Nuttall, “Some integrals involving the q-function,” *Naval Underwater Systems Center, New London, CT, Tech. Rep. 4297*, 1972.

[32] O. Simeone, O. Sahin, and E. Erkip, “Interference channel aided by an infrastructure relay,” in *Proc. IEEE Int. Symp. Information Theory*, Jul. 2009, pp. 2023–2027.

[33] O. Sahin, O. Simeone, and E. Erkip, “Gaussian interference channel aided by a relay with Out-of-Band reception and In-Band transmission,” *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 2976–2981, Nov. 2011.

[34] R. T. Rockafellar, *Convex Analysis*. Princeton University Press, 1970.